A boundedly invertible, $Q$ bounded does not imply $AQ$ densely defined.

Here we give an example of closed, boundedly invertible operator $A$ and a bounded operator $Q$ for which $AQ$ on its natural domain is not densely defined.

**Example**

Take on the Hilbert space $L^2(0,1)$ the operator $A$

$$Af = \frac{df}{d\zeta}$$
on the domain

$$D(A) = \{f \in L^2(0,1) \mid f \text{ is absolutely continuous}, \frac{df}{d\zeta} \in L^2(0,1) \text{ and } f(1) = 0\}.$$  

It is easy to see that $A$ is densely defined, and $A - \lambda I$ is boundedly invertible for all $\lambda \in \mathbb{C}$. Thus in particular, $A$ is closed.

As $Q$ we choose

$$(Qf)(\zeta) = \int_0^\zeta f(\tau)d\tau.$$  

Then it is easy to see that

$$D(AQ) := \{f \in L^2(0,1) \mid Qf \in D(A)\} = \{f \in L^2(0,1) \mid \int_0^\zeta f(\tau)d\tau \text{ is absolutely continuous,}$$ 

$$\frac{d}{d\zeta} \left[\int_0^\zeta f(\tau)d\tau\right] \in L^2(0,1) \text{ and } \int_0^1 f(\tau)d\tau = 0\} = \{f \in L^2(0,1) \mid \int_0^1 f(\tau)d\tau = 0\}.$$  

Thus every $f \in D(AQ)$ is orthogonal to the constant function, and so we see that this domain is not dense.