Exercises Nr.6: due, December 5

19-11-2011

<u>Ex.11.n1</u> ***['trust region method']***
Consider the quadratic Taylor approximation of f near x_k:

$$\begin{aligned} q(x) \quad := \quad & f(x_k) + \nabla f(x_k)^T (x - x_k) \\ & + \tfrac{1}{2}(x - x_k)^T \nabla^2 f(x_k)(x - x_k) \end{aligned}$$

Compute the descent step d_k according to $(*)$ *(Levenberg-Marquardt)* **and put $x_{k+1} = x_k + d_k$, $\tau := \|d_k\|$. Show that x_{k+1} is a local minimizer of the *trust region problem*:**

$$\min \; q(x) \quad \text{s.t.} \quad \|x - x_k\| \leq \tau$$

HINT: Show that the sufficient optimality conditions of Th.12.6 are satisfied.

Ex. **Solve the program:**

$$\min\ x^2 \text{ s.t. } g(x) := 1 - x \leq 0$$

by applying the penalty method with $p(x) = (g^+(x))^2$ and by applying the exact penalty method, with $p(x) = g^+(x)$.

Ex.6.2 **Show $X - xx^T \succeq 0 \Leftrightarrow \begin{pmatrix} 1 & x^T \\ x & X \end{pmatrix} \succeq 0$ and this (latter) constraint is convex in (x, X).**

Hint: Consider the condition
$(\alpha, y)^T \begin{pmatrix} 1 & x^T \\ x & X \end{pmatrix} \begin{pmatrix} \alpha \\ y \end{pmatrix} \geq 0\ \forall a \in \mathbb{R}, y \in \mathbb{R}^n$