Exercises Nr.5: due, Nov. 28

10-11-2011

Ex.11.6 Given the quadratic function on $\mathbb{R}^n$,
$q(x) = x^T Qx + b^T x$ $Q \succ 0$. Show that the minimizer t_k of

$$\min_{t \geq 0} q(x_k + td_k) \quad \text{is given by } t_k = -\frac{g_k^T d_k}{2 d_k^T Q d_k}.$$

Ex.11.9. Apply the steepest descent method (*i.e., with* $d_k = -g_k$) to

$$q(x) = x^T \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix} x \quad , \quad r \geq 1$$

Then with $x_0 = (r, 1)$ it follows

$$x_k = \left(\frac{r-1}{r+1} \right)^k (r, (-1)^k) \ .$$

(Linear convergence with factor $C = \frac{r-1}{r+1}$.)
Hint: Use Ex. 11.6 and induction wrt. k.

Ex.12.n1 **Consider the constrained program (P) with convex g_j's. Show that the following are equivalent:**
(a) There exists a point $\overline{x} \in \mathcal{F}$ at which MFCQ holds.
(b) The Slater condition holds, i.e., there exists x^* such that $g_j(x^*) < 0 \forall j \in J$.
(c) At all point $\overline{x} \in \mathcal{F}$ MFCQ holds.

Ex.12.11. **Show that LICQ implies MFCQ.**
HINT: Write the MFCQ condition in form of a system of equations.