Exercises Nr.4: due, Nov. 14

31-10-2011

Ex. 3.0' **Show that the feasible set of the primal and dual semidefinite program is a convex set. (***[see: 3.5 Semidefinite programming, sheets of Chapter 3].*

Ex. 3.1'

(a) Show that for $0 \neq a \in \mathbb{R}^n$ **the matrix** $A = aa^T$ **is positive semidefinite and has rank 1.**

(b) Let $A, B \in \mathbb{R}^{n \times n}$ **be matrices with columns** a_j**, rows** b_j^T**,** $j = 1, \ldots, n$**, respectively. Show that the following representation is true:**

$$A \cdot B = \sum_{j=1}^{n} a_j b_j^T \ .$$

(c) Let S, Z **be positive semidefinite. Then** $S \bullet Z \geq 0$ **and the equality holds if and only if** $S \cdot Z = 0$**.**

Hint: Use the Exercise (2) [see: 3.5 Semidefinite programming, sheets of Chapter 3].

Ex. 3.3' **(a) Let $u \in \mathbb{R}$, $a \in \mathbb{R}^{n-1}$ and let $I := I_{n-1}$ be the $(n-1) \times (n-1)$ unit matrix. Show:**

$$A := \begin{pmatrix} u & a^T \\ a & u \cdot I \end{pmatrix} \text{ is psd iff } u \geq \sqrt{a^T a}$$

(b) Show that the feasibility condition $x_n \geq \sqrt{\sum_{i=1}^{n-1} x_i^2}$ for the Lorentz (ice-cream) cone $\mathbb{L}_+^n$ can be written as the feasibility condition of an SDP program:

$$-A_0 + \sum_{k=1}^{n} A_k x_k \succeq 0$$

with certain matrices A_k.

Hint (a): Consider the condition
$(\alpha, x)^T A\binom{\alpha}{x} \geq 0 \ \forall a \in \mathbb{R}, x \in \mathbb{R}^{n-1}$