Exercises Nr.3:

30-10-2011

Ex.2.10: For the feasible set $\mathcal{F}$ of the program (CO) show that an ideal Slater point of $\mathcal{F}$ is in the relative interior of $\mathcal{F}$.

Ex. (*without Slater condition*) **(Under condition $C = \mathbb{R}^n, g_j \in C^1$) show that the pair $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}})$ $(\overline{x} \in \mathcal{F}, \overline{y} \geq 0)$ satisfies the KKT condition if and only if $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}})$ $(\overline{y} \geq 0)$ is a saddle point of $\boldsymbol{L}$.**

Hint: Use (proofs) of Th, 2.30, Cor. 2.33.

Ex2.3n. Farkas 2

(a) Let $g_j,\ j = 1, \ldots, m$, be convex functions on the convex set $C \subset \mathbb{R}^n$. Then, precisely one of the following alternatives I or II is true:

I. There exist a solution $x \in C$ of the system

$$g_j(x) < 0, \quad j = 1, \ldots, m$$

II. There exist $y_j \geq 0, j = 1, \ldots, m$ such that $\sum_{j=1}^m y_j = 1$ and

$$\sum_{j=1}^{m} y_j g_j(x) \geq 0 \quad \forall x \in C.$$

(b) For the linear case, *i.e.*, $g_j(x) = a_j^T x$, (a) means that precisely one of the following alternatives I or II is solvable

I. *There is a solution $x \in \mathbb{R}^n$ of:* $a_j^T x < 0, \quad j = 1, \ldots, m.$

II. *There is a solution $y_j \geq 0$ of:* $\sum_{j=1}^m y_j a_j = 0, \ \ \sum_{j=1}^m y_j = 1$.

(*Hint for (a): translate the condition (I) to the condition (1) in Farkas' Lemma 2.25***.)**