Comparing adaptive poleplacement with model reference

Explore the similarities and differences in implementation and response of the two classical adaptive algorithms when applied to the same control problem. Both theoretical as well as practical issues are to be explored. The problems are open ended. They are intended to build some intuition about the behaviour of adaptive control. They alert you to the practicalities involved in adaptive control design.

A control engineer has been given the task to achieve good tracking of low orbiting sattelites with a large communication antenna. The communication requirements dictate that the tracking error has to be less than 1% after transients.

The antenna structure can be modelled as:

\[(\sigma - 1)^2(\sigma^2 + 2p\cos(\theta)\sigma + p^2\cos(\theta))y = gu\]  

The signal \(u\) is the input (motor torque) and \(y\) is the output (antenna azimuth position). Both input and output are scalar variables. The parameter \(p\) indicates the amount of damping (or lack thereof) in the resonance of the structure. The parameter \(\theta\) is related to the natural resonance frequency of the antenna. \(g\) is related to the inertia of the antenna reflected through the gearbox of the motor.

The following information is available about the system model: \(p \in (0.95, 1.05), \theta \in (\pi/3, \pi/2)\) and \(g = 1\).

The reference trajectories that are to be tracked satisfy:

\[(\sigma - 1)^2r = 0\]  

Because the reference trajectories are part of the natural response of the system it suffices to stabilise the closed loop. (This follows from the internal model principle, see text Chapter 5.)

Also observe that:

\[(\sigma - 1)^2(\sigma^2 + 2p\cos(\theta)\sigma + p^2\cos(\theta))(y - r) = gu\]  

It is desired that the closed loop response corresponds to eigenvalues with modulus at most 0.8.
Adaptive pole placement

1. Propose an adaptive poleplacement strategy, using a plant representation that enables one to identify as few parameters as possible. (Hint: design the adaptive poleplacement as if there was no tracking requirement, then use $y - r$ as feedback signal in stead of $y$.)

2. Compare your design with the standard adaptive algorithm which would be based on a 4th order system model. Advantages? Disadvantages?

3. Implement your proposal and simulate a few responses for different initial conditions and reference trajectories.

4. Do the parameters converge to the desired values?

5. Is poleplacement achieved along the trajectories?

6. How well is the reference tracking achieved?

7. Suppose the actual reference signal satisfies the equation $(\sigma^2 - 1.9 * \sigma + 1)r = 0$. What happens? Is the tracking error acceptable?

Adaptive model reference control

1. Propose an adaptive model reference control strategy. Compute the model reference controller as a function of the plant parameters $p, \theta$. Is it possible to use this information to achieve a simpler tuning algorithm? (Hint: use as reference model $E(\sigma)y_r = v$ with $v$ derived from $r$ by a fixed transfer function that ensures that $y_r = r$ up to transients.)

2. Implement your algorithm and simulate for the same reference trajectories and plant initial conditions as above. Compare the response with the adaptive poleplacement control results. Which is to be preferred?

3. Which algorithm is the more complex?

4. Do the parameter estimates converge to the desired values?

5. Is tracking achieved? What is the difference in how tracking is achieved between the adaptive poleplacement and model reference strategy?

6. Suppose the actual reference signal satisfies the equation $(\sigma^2 - 1.9 * \sigma + 1)r = 0$. What happens? Is the tracking error acceptable? Explain. How could you improve the design in this case?