

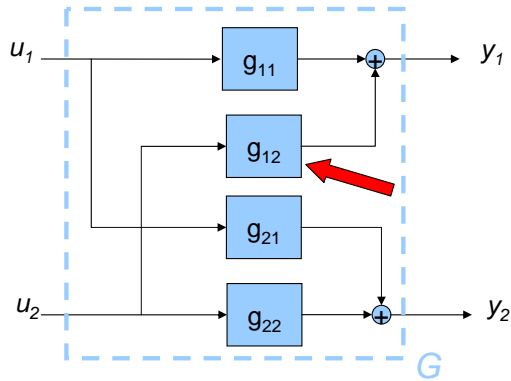
Interaction and Decoupling

Outline:

- Types of interaction
- Scaling
- Interaction measures
- Decoupling

In this module, we discuss interaction of plant dynamics. We illustrate some types of interaction and discuss the effect of I/O scaling. We derive tools to quantify interaction which will be used later in the exercises.

What is interaction?



$$y_1 = g_{11}u_1 + g_{12}u_2$$

$$y_2 = g_{21}u_1 + g_{22}u_2$$

$$\rightarrow y_1 = f(u_1, u_2)$$

$$y_1 \approx f(u_1) \text{ decoupled}$$

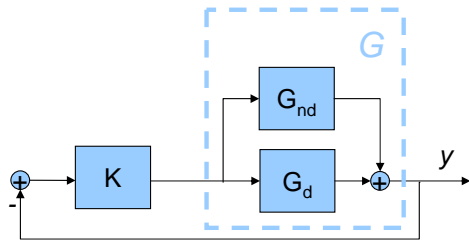
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When an output of a MIMO system is a function of more than one input, there is interaction. So if the plant is diagonal, there is no interaction. In this case, we call the system decoupled.

When a system is decoupled, we can make independent (decentralized) control designs. Decentralized control has fewer control parameters is easier to understand (and to retune). The requirement of independent control can lead to conservative designs. This conservatism can be reduced when decoupling techniques are used (discussed in module 8). Otherwise, we can also do sequential control design (non-independent control design), which will be discussed in module 11.

Why is it bad?



$$G = G_d + G_{nd}$$

$$T_d = G_d K_d (I + G_d K_d)^{-1}$$

$$E = G_{nd} G_d^{-1}$$

$$\underbrace{(I + GK)}_{\text{true MIMO}} = \underbrace{(I + ET_d)}_{\text{interaction analysis}} \underbrace{(I + G_d K_d)}_{\text{n loop SISO}}$$

$$\approx I$$

small if no interaction

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We can show this as follows:

The MIMO (interaction plant) can be seen as the sum of “the plant with only diagonal terms” and “the plant with everything but the diagonal terms”.

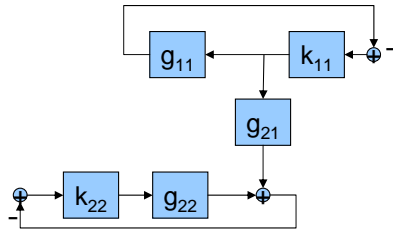
When we undertake decentralized control (diagonal controller terms only): we only consider the diagonal terms of the plant. This approximation is only ok when interaction is small: the diagonal terms G_{nd} relative to the diagonal terms G_d must be small.

One approach is to find measures for smallness of “E”, this is done in e.g. [Alb04, p.136]. Our experience is that these bounds are rather conservative for motion systems. Less conservative bounds are used in [Gros86] but are beyond the scope of this course. We will discuss two measures for interaction which have been proven to be useful in practical situations.

[Gros86] Interaction measures for systems under decentralized control
Automatica, Volume 22, Issue 3, May 1986, Pages 309-319
Pierre Grosdidier and Manfred Morari

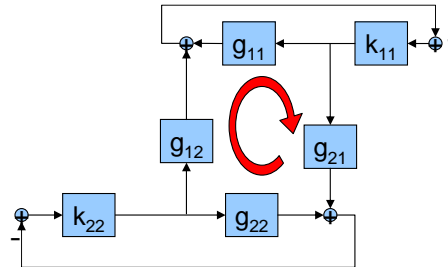
Types of interaction

One way (interference)



- no stability problem
- performance (disturbance)

two way (coupling)



- stability problem!
- performance (disturbance)

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We distinguish two types of interaction:

one way (interference) interaction:

- Can be seen as a kind of feedforward/disturbance from one loop to the other
- The influence of this one way interaction is smaller when the “loop 2” has much higher bandwidth than “loop 1”
- The openloop transferfunction matrix has triangular form.
- Recall how you calculated eigenvalues of a triangular matrix

Two way interaction: coupling

- Can be seen as an “underwater” feedback loop, so that stability problems may occur when this kind of interaction is neglected.
- Just a higher bandwidth may not always be the solution in this case (see example Module 7).
- The openloop transferfunction matrix has full matrix form.
- Alb04: p. 126

Scaling

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$T_{so} = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix}, \quad T_{si} = \begin{bmatrix} 1 & 0 \\ 0 & \beta \end{bmatrix}$$

Q1: what is $G_{scaled} = T_{so}GT_{si}$

Q2: what happens when $\alpha\beta = 1$, $\alpha \ll \beta$

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If we want to say something about interaction: why not just look at the off-diagonal terms and see if they are small enough?

Answer: scaling! We illustrate the effect of scaling on a control problem.

I/O scaling is often very arbitrary in industrial applications; how to compare rotations with translations?

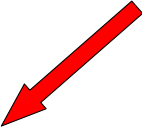
Q1: Interaction: how large are the off-diagonal terms?

Q2: “

We see that scaling can trouble a design and makes statements about interaction very different. There is no general solution to the scaling problem: a commonly used approach is to scale the I/O so that signals of channels are of “comparable magnitude”, e.g. all signals scaled to “1”: e.g. divide the variables though the maximum magnitude one can expect... This is a pretty messy approach.

Sko96: p. 5, 161

3x3 scaling

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \quad T_{so} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix}$$


$$T_{si} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \delta \end{bmatrix}$$

$$G_{scaled} = \begin{bmatrix} g_{11} & \gamma g_{12} & \delta g_{13} \\ \alpha g_{21} & \alpha \gamma g_{22} & \delta \alpha g_{23} \\ \beta g_{31} & \beta \gamma g_{32} & \delta \beta g_{33} \end{bmatrix} \quad \text{Hmm...}$$

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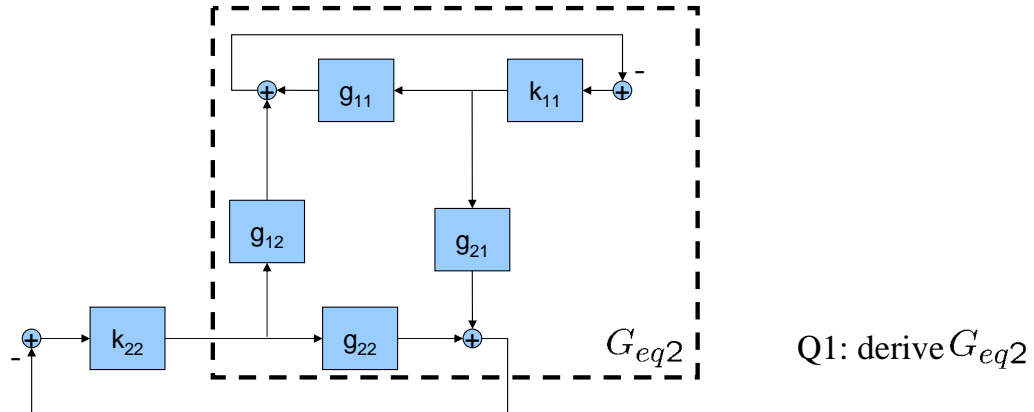
For large systems, scaling can be pretty nasty...

See that output scaling has effect on the rows while input scaling has effect on the columns.

You, as an engineer, are to solve this “very practical” problem as academics decided that it is solvable.

Lets keep this scaling problem in mind and continue with measures of interaction...

Interaction index



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Back to the problem: we want to design k_{11} and k_{22} (decentralized controller) for a system with coupling (two way interaction). How independent can we design k_{22} ?

Q1: Say we already closed the first loop, what is G_{eq2} (the plant as seen by the controller k_{22})?

Interaction index

$$\begin{aligned}
 G_{eq2} &= g_{22} \overbrace{-\frac{g_{12}k_{11}g_{21}}{1 + g_{11}k_{11}}}^{\text{Due to interaction}} \\
 &= g_{22} \left(1 - \underbrace{\frac{g_{12}g_{21}}{g_{11}g_{22}}}_{\Phi} \frac{g_{11}k_{11}}{1 + g_{11}k_{11}} \right) = g_{22}(1 - \Phi t_{11})
 \end{aligned}$$

Interaction index
 $\Phi = \frac{g_{12}g_{21}}{g_{11}g_{22}} \approx 0$

Relative gain
 $\lambda = \frac{1}{1 - \Phi} \approx 1$

}

No interaction!

The plant as seen by controller k_{22} does not only contain g_{22} , but also elements of the other loop (closed by k_{11}). We work this out as follows.

We define the term ϕ as the interaction index.

And we define the relative gain (which will be used later)

We see that if the interaction index is small (zero), there is less (no) interaction,

the relative gain equals one if interaction is small

If there is interaction, control design (k_{22}) can not be done considering only the diagonal elements of the plant (g_{22} in this case).

Interaction index and scaling

Q1: derive ϕ for

$$G_{scaled} = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix} G \begin{bmatrix} 1 & 0 \\ 0 & \beta \end{bmatrix}$$
$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

What is the effect of scaling on the interaction index?

Relative gain array (RGA)

$$\text{relative gain } \Lambda_{ii} = \frac{1}{1 - \Phi}$$

- nxn systems

- scaling insensitive

$$\text{relative gain array } \Lambda(G) = G \star (G^{-1})^T \quad \bullet \text{ only two way interaction}$$



element wise: Matlab `.*`

$$\Lambda(G) = \frac{1}{g_{11}g_{22} - g_{12}g_{21}} \begin{bmatrix} g_{11}g_{22} & -g_{12}g_{21} \\ -g_{12}g_{21} & g_{11}g_{22} \end{bmatrix} = \begin{bmatrix} \Lambda_{ii} & 1 - \Lambda_{ii} \\ 1 - \Lambda_{ii} & \Lambda_{ii} \end{bmatrix}$$

$$\Lambda(G(j\omega)) \approx I \quad \text{no interaction!}$$

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A more general, which can also be defined for systems larger than 2x2 is the relative gain array (RGA). A general expression for nxn systems is given here. Mind the element wise multiplication “.*”. When the RGA equals the identity matrix for the frequency range of interest, the system is decoupled at these frequencies (there is no two way interaction). If it is not, we can not conclude that the system is decoupled, we still have no idea how large the interaction is in meters/rad/Kelvin..., but we know that we are in trouble.

Alb04. p. 133.

Sko96. p. 88, 434

Relative gain array (RGA)

$$\Lambda(G) = G \star (G^{-1})^T$$

- nxn systems
- scaling insensitive
- rows/columns sum up to one

Q: Calculate $\Lambda(G)$ for

$$G = \begin{bmatrix} g_{11} & g_{12} \\ 0 & g_{22} \end{bmatrix}$$

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The rows of the RGA sum up to one

The columns of the RGA sum up to one

The RGA does not depend on the dimensions (or scaling) of the plant

The RGA can be used to find pairings (useful combinations: which loops to close) of I/O's Sko96 p.439

Many interpretations (e.g. relations to NMP-zeros, stability, conditioning) of the RGA at DC (frequency 0) exists, Sko96 p.88.

Alb04. p. 133.

Sko96. p. 88, 434

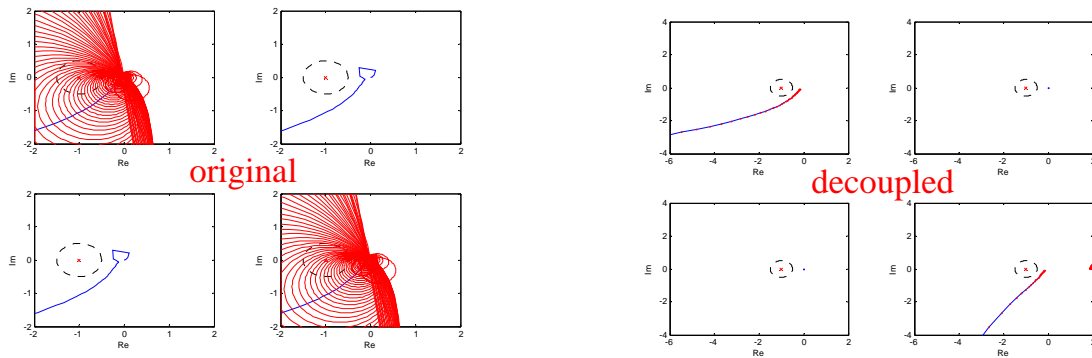
Static decoupling

- why?
- how? decoupling examples
- what happens
- experiments

In this part we discuss the following issues.

Why?

$$\underbrace{(I + GK)}_{\text{true MIMO}} = \underbrace{(I + ET_d)}_{\text{interaction analysis}} \underbrace{(I + G_d K_d)}_{\text{n loop SISO}}$$



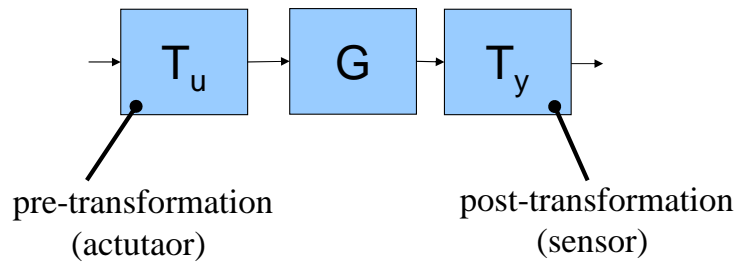
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- As we studied earlier, interaction tells us how far we are from a true MIMO system when we only consider the diagonal terms of the plant (decentralized control). If interaction is large, we must be very careful in the design of a diagonal controller.
- To illustrate this, we plotted the nyquist array for the experimental system (the setup we use in this course). The red circles are a measure for the uncertainty we have to take in account when we design n SISO controllers (decentralized control), [Alb04,p 63: *Gershgorin radius*]. The radius of these circles are closely related to the size of the interaction index we studied earlier. We see that for this system in its original setting, it is nearly impossible to achieve satisfactory performance.
- By I/O transformations we can change the plant, we already experienced the power of this in the SIMO/MISO experiments.
- In the right figure, we have the same plants with a specific I/O transformation so that the new plant is diagonal: if the plant is diagonal: interaction is small and we have a very strong relation between “n-loop SISO” and “true MIMO” (we have no uncertainty: smaller red circles).

Why?

- transform to decoupled form -> independent SISO design
- change of control variables (+ or -)
- possible interaction in other transferfunctions

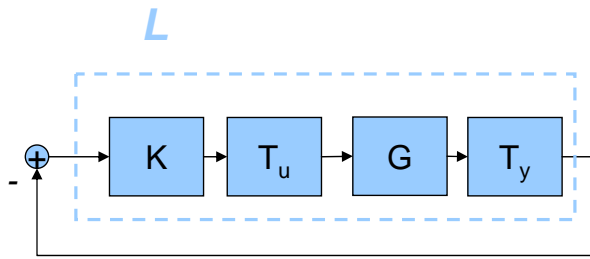


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- An I/O transformation changes the plant... this has a lot of implications!
- We have less controllers to design n instead of $n \times n$
- The controllers can be tuned independently: strong link to SISO control design.
- A part of the plant can still be stable when one loop fails
- Sometimes it is easier to design controllers directly on the to be controlled variables instead of directly on sensor outputs. Sometimes not.
- Openloop decoupled does not mean that other transfers (e.g. sensitivity) are decoupled as well, we will study that later

Why?



T_y mixes sensors

T_u mixes actuators

$$L = \underbrace{T_y G T_u}_{\text{diagonal}} \underbrace{K}_{\text{diagonal}}$$

$$L = \begin{bmatrix} l_{11} & 0 \\ 0 & l_{22} \end{bmatrix}$$

Red arrows point from k_{11} to l_{11} and from k_{22} to l_{22} .

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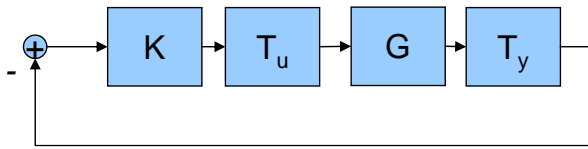
When we use post and pre transformations of the plant, we may be able to make the openloop (transformed plant times controller) diagonal: decoupled.

A post transformation mixes the sensors.

A pre transformation mixes the actuators.


As a result, the closed loop (in the new coordinates) is diagonal (decoupled). This means that one diagonal term of the open-loop depends on one diagonal term of the controller.

Why?



$$L = \begin{bmatrix} l_{11} & 0 \\ 0 & l_{22} \end{bmatrix}$$

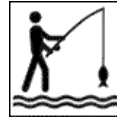
$$T = L(I + L)^{-1}$$

Stability  Q1: what are the eigenvalues of the closed loop?

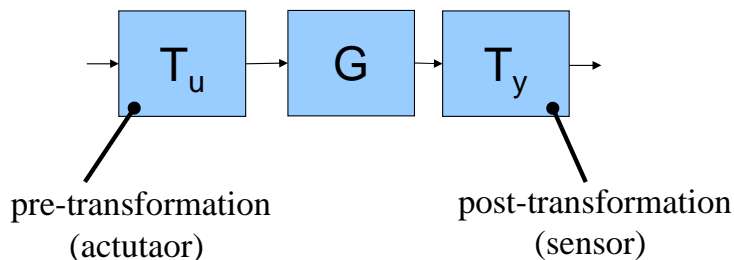
If we have a diagonal openloop transfer, what happens with the (non diagonal terms) of the closed loop transfer?

How?

- only static decoupling
- T_u T_y decoupling problem hard!
- Use (engineering?) creativity!



- geometry
- scalar dynamics
- symmetry



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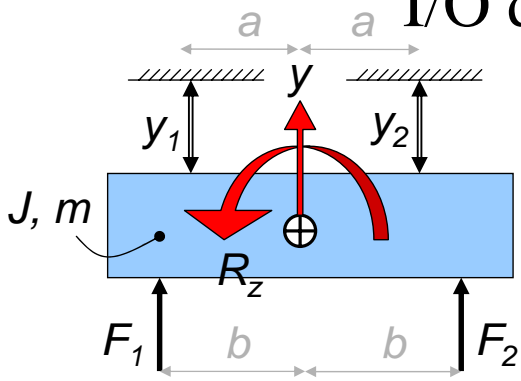
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- We now continue with decoupling of the plant.
- We focus on static decoupling only: dynamic decoupling is often very difficult because of plant uncertainty: requires a detailed model: we then prefer to do model based control design: Module12.
- There is one problem: solving I and O transformations at the same time is very hard : non-convex, for one (T_u or T_y) at the time, there are (numerical) tools [Munro04], but this is beyond the scope of this course. Numerical solutions are still ad hoc and require a lot of (numerical) luck!
- When you find a decoupling transformation it is really worth it! So in practice, you need a lot of creativity to find decoupling transformations: often based on physical insight. In this module, we give three examples to inspire you.

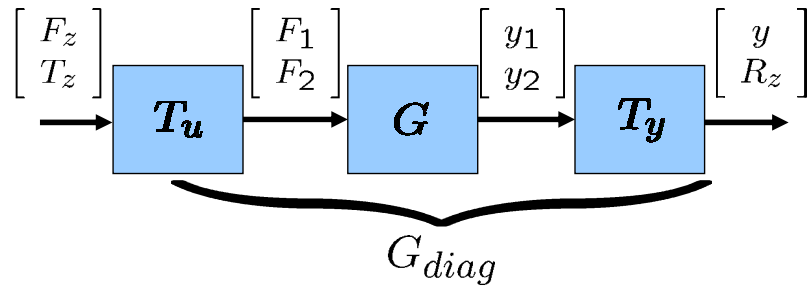
The best decoupling transformations are those, where the physical insight to the plant dynamics is preserved. We discuss geometric decoupling, decoupling based on scalar dynamics and (reflective) symmetry.

[Munro04] "Diagonal dominance using LMIs", Chughtai, S.S.; Munro, N.; Control Theory and Applications, IEE Proceedings- Volume 151, Issue 2, 23 March 2004 Page(s):225 - 233

I/O decoupling: geometry



Q: Derive T_u T_y so that plant is decoupled in coordinates y, R_z (small angles only)



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We illustrate how geometry can be used to decouple a system.

Given the beam-system in this figure. We have two rigid body modes, two actuators, two sensors. If the actuators and sensors are located in this way, interaction is present: the non-diagonal terms of G are nonzero.

Q: derive (using geometric relations) input output transformations so that we can control the two rigid body modes independently: that is G^{diag} is decoupled. Consider small angles only.

I/O decoupling: scalar dynamics

Plant has special structure so that;

$$G(s) = g(s) \underbrace{\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}}_{\tilde{G}}$$

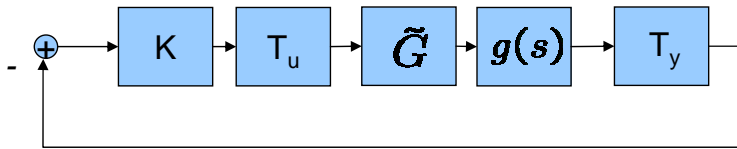
$$g(s)\tilde{G} = \tilde{G}g(s)$$

choose

$$T_y \tilde{G} = I$$

or

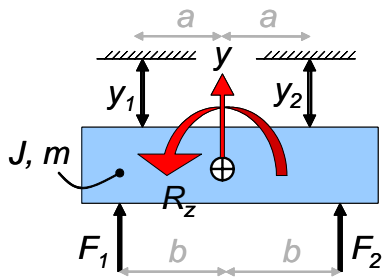
$$\tilde{G} T_u = I$$



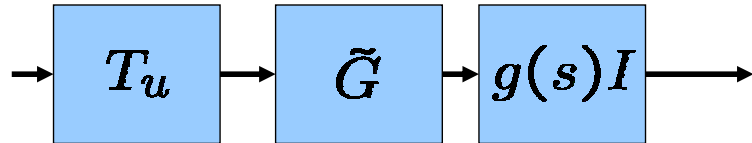
Q: which transformation should we prefer?

Here we have a plant with scalar dynamics: this is for example the case if a system has dominant rigid body modes (the flexible modes can then be considered as residual interacting dynamics: uncertainty). We see that we can decouple the plant with pre or post compensation; which one would you prefer?

I/O decoupling: scalar dynamics



Q1: Derive only T_u so that the open-loop is decoupled



$$G = \underbrace{\frac{1}{Jms^2}}_{g(s)} \underbrace{\begin{bmatrix} J + abm & (J - abm) \\ (J - abm) & (J + abm) \end{bmatrix}}_{\tilde{G}}$$

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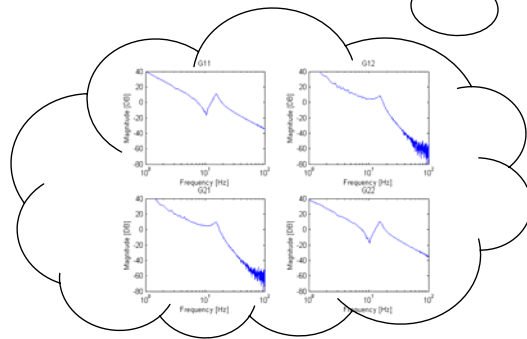
For the example: calculate the input decoupling matrix: how does the new plant look like? Can you give an interpretation?

Discussion: What is the difference with the decoupling procedure using geometry?

I/O decoupling: symmetry

- Plant has special structure so that;

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} G = G \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow G = \begin{bmatrix} G_1 & G_2 \\ G_2 & G_1 \end{bmatrix} \quad (1)$$

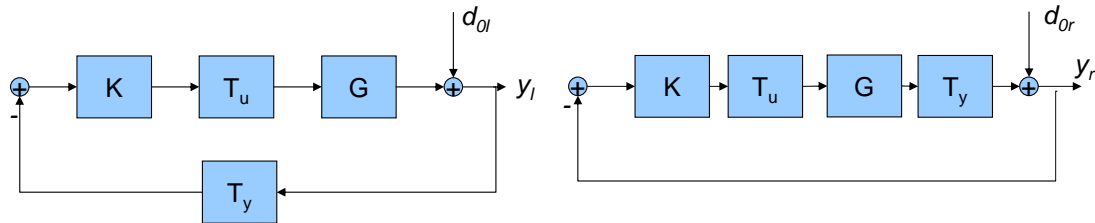


Q1: What are the eigenvectors and eigenvalues of G ?

Q2: decouple G

Some systems have reflectional symmetry: this is a stronger condition than symmetry. Mechanical systems where mechanical modes are actuated and controlled at both sides of the flexibility are (partially) reflectional symmetric: so is the experimental setup. These systems can be decoupled using static pre and post compensations. Find the eigenvectors and the eigenvalues of this G . Use this knowledge to decouple the plant.

What happens?



$$L = \underbrace{T_y G T_u}_{\text{diagonal}} \underbrace{K}_{\text{diagonal}}$$

Q: What is the transfer from d_0 to y in both cases?

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When we introduce pre and post multiplication of the plant, we change the controlled variables of the plant. The objective is to create a decoupled open-loop as was discussed earlier. Other transfers may not be decoupled as they are expressed in other coordinates.

To see this, derive the following two transfers. In the left figure we have the physical plant output disturbance and the true physical measured variable y .

Q: Derive the transfer from d_0 to y .

In the left figure we have another disturbance and a measured variable y (both after sensor transformation). Q: Derive the transfer from d_0 to y .

Q: Note that d_0 and y in both figures are not the same! Derive the relation between d_{0r}, d_{0l}

Q: What happens when we have a physical disturbance with direction $[1 \ 0]$, $[-1 \ 1]$ and $[1 \ 1]$?

So: decoupling of the open-loop may not lead to disturbance decoupling: and may even lead to coupling of physical disturbances: they appear in loops where you do not want them!

MIMO design approach

- measure/identify the mimo FRF data
- Use interaction measures to assess the amount of interaction
- Investigate decoupling
- Investigate sequential loop closing
- Use model based design