

Answers exercises Extension chapter 7 (Joint continuous distributions)

8. a., b. $\frac{3}{8}, \frac{1}{4}, 0$ c. $\frac{1}{2}$
9. a. $c = 4$ b. $f_X(x) = f_Y(x) = 2e^{-2x}$ ($x \geq 0$)
 c. 1.83%, 25.24%, 0.5, 0 d. $F_{X,Y}(x, y) = (1 - e^{-2x})(1 - e^{-2y})$ for $x, y \geq 0$ (and = 0 elsewhere)
10. a. $f_X(x) = 4x - 4x^3$ if $0 < x < 1$, $f_Y(y) = 4y^3$ if $0 < y < 1$.
 b. $E(X) = \frac{8}{15}$, $var(X) = \frac{11}{225}$, $E(Y) = \frac{4}{5}$, $var(Y) = \frac{2}{15}$, $E(XY) = \frac{4}{9}$, $\rho(X, Y) = 4/\sqrt{66}$.
11. a. see 10a.
13. a. $c = \frac{1}{\pi}$ b. $f_X(x) = f_Y(x) = \frac{2}{\pi}\sqrt{1-x^2}$, if $-1 < x < 1$
 c. No d. $f_X(x|Y=y) = \frac{1}{2\sqrt{1-y^2}}$, if $|x| < \sqrt{1-y^2}$ e. 0
14. a. $f_{X,Y}(x, y) = \frac{3}{4}e^{-3u}$, if $0 \leq x \leq 4$ and $y \geq 0$. b. $\frac{1}{4}e^{-3}$ c. $\frac{3}{25}$
15. a. $f_Z(z) = 2e^{-z}(1 - e^{-z})$, if $z > 0$ b. 1.5, 1.25 c. $\frac{1}{3}$ d. $\frac{1}{5}\sqrt{5}$, $\frac{2}{5}\sqrt{5}$
16. a. $f_{X_1, \dots, X_n}(x_1, \dots, x_n) = \lambda^n e^{-\lambda \sum_{i=1}^n x_i}$ b. $f_W(w) = n\lambda(1 - e^{-\lambda w})^{n-1} e^{-\lambda w}$, if $w > 0$.