# Trial test Calc. I – Functions, Limits, Continuity and Derivatives – April 2018

**1.** Given is the function $f(x) = \frac{x+1}{1-x}$

**a.** Find the inverse of $f$.

**b.** State the domain and the range of $f$.

**c.** Find all asymptotes of $f$.

**d.** If $g(x) = 2x - x^2$, determine $f \circ g(x)$ and compute $\lim_{x\to-\infty} f \circ g(x)$.

**2.** Evaluate the limit if it exists: if it does not exist show why not.

**a.** $\lim_{x\to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

**b.** $\lim_{x\to 1} \dfrac{x - 1}{2x - \sqrt{3 + x^2}}$

**c.** $\lim_{x\downarrow 2} \dfrac{x^2 - 9x + 14}{2x^2 - 4x}$

**d.** $\lim_{x\to 1} \dfrac{e^x - 1}{x^2}$

**e.** $\lim_{x\to\infty} \dfrac{1 - e^x}{1 + 2e^x}$

**f.** $\lim_{x\to 0} \dfrac{(x+3)^{-1} - 3^{-1}}{x}$

**3.** Consider the function $f(x) = \sqrt{x}$

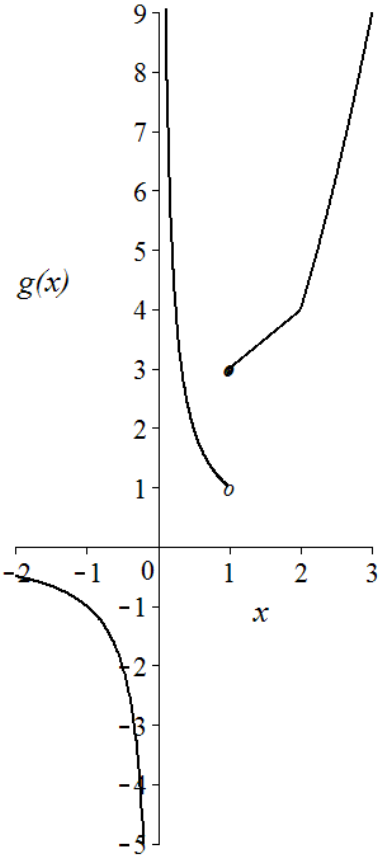
**a.** Show with the definition of derivative that $f'(x) = \frac{1}{2\sqrt{x}}$

**b.** Find the slope and the equation of the tangent line at the point (4, 2) of the graph of $f$.

**c.** Is $f$ continuous and differentiable at all values of its domain? Explain why (not).

**4.** Consider the function $g(x) = \begin{cases} \frac{1}{x}, & if\ x < 1 \\ x + 2, & if\ 1 \le x \le 2 \\ x^2, & if\ x > 2 \end{cases}$

which is graphed as well.




**a.** State the domain of the function $g$.

**b.** Find the following limits, if they exist.
(*Use the graph only as a check*)

1. $\lim_{x\uparrow 0} g(x)$
2. $\lim_{x\downarrow 1} g(x)$
3. $\lim_{x\uparrow 1} g(x)$
4. $\lim_{x\to 1} g(x)$
5. $\lim_{x\to-\infty} g(x)$

**c.** At which numbers is the function discontinuous? Motivate your answer and state the nature of the discontinuity at these numbers.

**d.** Is the function $g$ differentiable at $x = 2$? Why (not)? *Motivate your answer using limits!*