

Intermediate test Applied Statistics 29th of April 2018 (Ch 2-4), 14.30-15.30

A formula sheet and probability tables are provided. Write name and math number on your sheets

1. After complaints about alleged slowness of the computer network the data centre of “Stork Ketels” decided to investigate the response time of some heavy use program commands. They observed 16 waiting times for these commands on arbitrarily chosen moments during working hours: the mean response time was 15.10 seconds and the sample standard deviation 5.06 seconds.
 - a. Determine a numerical 90%-confidence interval for the expected response time of such a command.
 - b. Determine a numerical 95%-confidence interval for the standard deviation of the response times.
 - c. Give the proper interpretation of the confidence interval you found in b.
 - d. Prior to the research the data centre stated that an expected response time of at most 13 seconds is acceptable. Does the sample provide sufficient evidence for the statement that the mean response time is higher than 13 seconds, at a 10% level significance? Conduct the proper test in 8 steps.

2. The exponential density function can be given as $f_X(x|\mu) = \frac{1}{\mu} e^{-x/\mu}$, where $\mu > 0$ is the unknown expectation. A realization x_1, \dots, x_n of the random sample X_1, \dots, X_n of X is available.
 - a. Show that \bar{X} is the maximum likelihood estimator (*mle*) of μ .
 - b. Determine the *mle* of $\text{var}(X)$ and check out whether this estimator is unbiased.
 - c. Determine the value of the constant c , such that $T = c \cdot \bar{X}$ the best estimator of μ
 - d. Use Neyman-Pierson’s fundamental lemma to find the most powerful test on $H_0: \mu = 10$ against $H_1: \mu = 8$ (for a given α).
 - e. Find (approximate) for the test in d. 1. the rejection region and
2. The power of the test if $n = 100$ and $\alpha = 5\%$
 - f. Show that the test in d. is also the uniformly most powerful test for $H_0: \mu = 10$ against $H_1: \mu < 10$ (for a given α).

Formula Sheet Applied Statistics

Probability Theory

$$E(X + Y) = E(X) + E(Y)$$

$$E(X - Y) = E(X) - E(Y)$$

$$E(aX + b) = aE(X) + b$$

$$\text{var}(X) = E(X^2) - (EX)^2$$

$$\text{var}(aX + b) = a^2 \text{var}(X)$$

If X and Y are independent:

$$\text{var}(X + Y) = \text{var}(X) + \text{var}(Y), \quad \text{var}(X - Y) = \text{var}(X) + \text{var}(Y)$$

Distribution	Probability/Density function	Range	$E(X)$	$\text{var}(X)$
Binomial (n, p)	$\binom{n}{x} p^x (1-p)^{n-x}$	$0, 1, 2, \dots, n$	np	$np(1-p)$
Geometric (p)	$(1-p)^{x-1} p$	$1, 2, 3, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson (μ)	$\frac{e^{-\mu} \mu^x}{x!}$	$0, 1, 2, \dots$	μ	μ
Uniform on (a, b)	$\frac{1}{b-a}$	$a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential (λ)	$\lambda e^{-\lambda x}$	$x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal (μ, σ^2)	$\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	\mathbb{R}	μ	σ^2

Testing procedure in 8 steps

1. Give a probability model of the observed values (the statistical assumptions).
2. State the null hypothesis and the alternative hypothesis, using parameters in the model.
3. Give the proper test statistic.
4. State the distribution of the test statistic if H_0 is true.
5. Compute (give) the observed value of the test statistic.
6. State the test and
 - a. Determine the rejection region
 - b. Compute the p-value.
7. State your statistical conclusion: reject or fail to reject H_0 at the given significance level.
8. Draw the conclusion in words.

Bounds for Confidence Intervals

$$\hat{p} \pm c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \Phi(c) = 1 - \frac{1}{2}\alpha$$

$$\bar{X} \pm c \frac{S}{\sqrt{n}}, \quad P(T_{n-1} \geq c) = \frac{1}{2}\alpha$$

$$\left(\frac{(n-1)S^2}{c_2}, \frac{(n-1)S^2}{c_1} \right),$$

$$P(\chi_{n-1}^2 \leq c_1) = P(\chi_{n-1}^2 \geq c_2) = \frac{1}{2}\alpha$$

Test statistics

X (binomial number of successes)

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

S^2

Prediction interval: $\bar{X} \pm c \sqrt{S^2 \left(1 + \frac{1}{n}\right)}$