

# Test Calculus I – Functions, Limits, Continuity and Derivatives – April 29, 2018

(Note that “ $\lim_{x \uparrow a}$ ” and “ $\lim_{x \rightarrow a^-}$ ” mean the same, and so do “ $\lim_{x \downarrow a}$ ” and “ $\lim_{x \rightarrow a^+}$ ”)

1. Given is the function  $f(x) = 2x - x^2$ 
  - a. Is  $f$  an even or an odd function?
  - b. Find the  $x$ -intercepts and the  $y$ -intercept of  $f$
  - c. Verify that  $f$  can be written as  $f(x) = -(x - 2)^2 + 4$ . Which transformations do we have to conduct successively to graph  $f$ , starting with the graph  $y = x^2$ ?
  - d. State the domain and the range of  $f$ .
  - e. If  $g(x) = \frac{1}{1-x}$ , find  $g \circ f(x)$  and state the domain of  $g \circ f$
  - f. Use the definition of the derivative to show that  $f'(x) = 2 - 2x$ .
  - g. Use the result in f. to find the slope and the equation of the tangent line of the graph of  $f$  for  $x = 0$ .
  
2. Evaluate the limit if it exists: if it does not exist show why not.

a.  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 - 10x + 25}$

e.  $\lim_{x \rightarrow \infty} \frac{e^{2x} - e^x}{1 + 2e^{2x}}$

b.  $\lim_{x \downarrow 5} \frac{x^2 - 25}{x^2 - 10x + 25}$

f.  $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x^2}$

c.  $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 - 10x + 25}$

g.  $\lim_{x \rightarrow 0} \frac{\cos(x)}{x^2}$

d.  $\lim_{x \rightarrow 4} \frac{x^2 + 4x - 32}{2 - \sqrt{x}}$

h.  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

3. Consider the function  $g(x) = \begin{cases} 2x, & \text{if } x < 0 \\ x^2, & \text{if } 0 < x \leq 1 \\ 2x - 1, & \text{if } x > 1 \end{cases}$

a. Find the following limits, if they exist.

1.  $\lim_{x \uparrow 0} g(x)$     2.  $\lim_{x \downarrow 0} g(x)$     3.  $\lim_{x \rightarrow 0} g(x)$     4.  $\lim_{x \downarrow 1} g(x)$     5.  $\lim_{x \uparrow 1} g(x)$

b. Is  $g$  discontinuous at  $x = 0$  and/or at  $x = 1$ ?

Motivate your answer and state the nature of the discontinuity at these numbers.

c. Is the function  $g$  differentiable at  $x = 0$ ? Why (not)?

d. Is the function  $g$  differentiable at  $x = 1$ ? Why (not)?

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Grade =  $1 + 9 \times \frac{\# \text{ of points}}{41}$

1							2	3				Total
a	b	c	d	e	f	g		a	b	c	d	
1	2	2	2	2	3	2	8 × 2	5	2	2	2	41