Study aid Probability 2 (the formulas on the formula sheet are given)

Definitions:

Discrete: $E(X) =$ **Continuous:** $E(X) =$	$cov(X, Y) =$
$var(X) =$	$\rho(X, Y) =$

Calculation formulas :

Discrete: $E(X^2) =$ **Continuous:** $E(X^2) =$	$var(X) =$
$E(XY) =$	$cov(X, Y) =$
$E(X \mid Y = y) =$	$E(X) = E[\qquad\qquad]$ $= \sum_{..} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$

Properties $E(X), var(X), cov(X, Y)$ and $\rho(X, Y)$:

$E(aX + b) =$	$var(aX + b) =$	
$E(X + Y) =$ $E(X - Y) =$	$var(X + Y) =$ $var(X - Y) =$	
$E(\sum_{i=1}^{n} X_i) =$	$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + \sum_{i}\sum_{\neq j} cov(X_i, X_j)$	
$cov(aX, Y) =$	$\rho(aX, Y) =$	$cov(X, X) =$
$cov(X, Y + Z) =$		

Independence

Discrete: X **and** Y **are independent** $\Leftrightarrow P(X = x \text{ and } Y = y) = \ldots\ldots\ldots\ldots\ldots\ldots$

General: X **and** Y **are independent** $\Leftrightarrow P(X \in A \text{ and } Y \in B) = \ldots\ldots\ldots\ldots\ldots\ldots$

If X and Y are independent, then we have:	$E(XY) =$ $cov(X, Y) =$ and $\rho(X, Y) =$ $var(X \pm Y) =$
If $X_1, \ldots, X_n$ are independent and identically distributed with expectation μ and variance σ^2, then:	$var(\sum_{i=1}^{n} X_i) =$ $E(\overline{X}) =$ *and* $var(\overline{X}) =$

Convolution-sum (Ch. 5): $P(X + Y = k) =$

Convolution-integral (Ch. 7): $f_{X+Y}(z) =$

***Study aid** 2*

Definitions:

Discrete: $E(X) = \sum_x xP(X=x)$ **Continuous:** $E(X) = \int_{-\infty}^{\infty} xf(x)dx$	$cov(X,Y) = E(X-EX)(Y-EY)$
$var(X) = E(X-EX)^2$	$\rho(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$

Calculation formulas:

Discrete: $E(X^2) = \sum_x x^2 P(X=x)$ **Continuous:** $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$	$var(X) = E(X^2) - (EX)^2$
$E(XY) = \sum_x \sum_y x \cdot y \cdot P(X = x \text{ and } Y = y)$	$cov(X,Y) = E(XY) - E(X)E(Y)$
$E(X\|Y=y) = \sum_x x \cdot P(X = x\|Y=y)$	$E(X) = E[\,E(X\|Y)]$ $= \sum_y E(X\|Y=y) \cdot P(Y=y)$

Properties $E(X), var(X), cov(X,Y)$ and $\rho(X,Y)$:

$E(aX+b) = aE(X) + b$	$var(aX + b) = a^2\, var(X)$	
$E(X+Y) = E(X) + E(Y)$ $E(X-Y) = E(X) - E(Y)$	$var(X+Y) = var(X) + var(Y) + 2cov(X,Y)$ $var(X-Y) = var(X) + var(Y) - 2cov(X,Y)$	
$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$	$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + \sum_{i \neq j}\sum cov(X_i, X_j)$	
$cov(aX,Y) = a \cdot cov(X,Y)$	$\rho(aX,Y) = \rho(X,Y), a > 0$	$cov(X,X) = var(X)$
$cov(X, Y+Z) = cov(X,Y) + cov(X,Z)$		

<u>Independence</u> Discrete:

***X* and *Y* are independent** $\Leftrightarrow P(X = x \text{ and } Y = y) = P(X=x) \cdot P(Y=y)$**,**

General:

***X* and *Y* are independent** $\Leftrightarrow P(X \in A \text{ and } Y \in B) = P(X \in A) \cdot P(Y \in B)$

If *X* and *Y* are independent, then we have:	$E(XY) = E(X)E(Y)$ $cov(X,Y) = 0$ and $\rho(X,Y) = 0$ $var(X \pm Y) = var(X) + var(Y)$
If $X_1, \ldots, X_n$ are independent and identically distributed with expectation μ and variance σ^2, then:	$var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} var(X_i) = n\sigma^2$ $E(\overline{X}) = \mu$ and $var(\overline{X}) = \frac{\sigma^2}{n}$

Convolution-sum: $P(X+Y=k) = \sum_k P(X=k) \cdot P(Y=n-k)$

Convolution-integral: $f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)dx$