Study aid 1: Common discrete distributions and their expectation and variance (Ch. 4)

Name	When to apply?	$P(X = k) =$	$E(X)$	$var(X)$	Example
Binomial $B(n, p)$					
Geometric (p)			$\frac{1}{p}$	$\frac{1-p}{p^2}$	
Hyper-geometric (R, N, n)			$n \cdot \frac{R}{N}$	$n\frac{R}{N}\left(1-\frac{R}{N}\right) \times \frac{N-n}{N-1}$	
Poisson (μ)		$\frac{\mu^k e^{-\mu}}{k!}$, $k = 0,1, \ldots$	μ	μ	
Homogeneous op $1, \ldots, n$				For given n apply $var(X) = E(X^2) - \mu^2$	

Relations between discrete distributions (approximations):

1.

2.

Name/ $par.$	**When to apply?**	$P(X=k)=$	$E(X)$	$var(X)$	**Example**
Binomial $B(n,p)$	Count the number of successes in n (ind.) Bernoulli trials with success probability p:	$\binom{n}{k}p^k(1-p)^{n-k}$, $k=0,1,\ldots,n$	np	$np(1-p)$	**X = number of sixes in 10 rolls with a dice. $X \sim B(10,\frac{1}{6})$**
Geometric (p)	Count the number of Bernoulli trials until a success is achieved.	$(1-p)^{k-1}\,p$, $k=1,2,\ldots.$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	**X = Number of rolls to get a first 6. X~geom$(1/6)$**
Hyper-geometric (R,N,n)	Count the number of successes in n draws without replacement from a population with R successes and $N-R$ failures	$\frac{\binom{R}{k}\binom{N-R}{n-k}}{\binom{N}{n}}$, $k=0,..,n$	$n\cdot\frac{R}{N}$	$n\frac{R}{N}\left(1-\frac{R}{N}\right)\times\frac{N-n}{N-1}$	**X = the number of boys if 4 persons are chosen from a group 5 boys and 7 girls.**
Poisson (μ)	Count the number of “rare” events in an area / interval of time	$\frac{\mu^k e^{-\mu}}{k!}$, $k=0,1,\ldots$	**μ**	**μ**	**X = the number of customers that enter a post office within 10 minutes.**
Homogeneous on $1,\ldots,n$	An arbitrary choice from the numbers $1,2,\ldots,n$	$\frac{1}{n}$, $k=1,\ldots,n$	$\frac{n+1}{2}$	**For given n apply** $var(X) = E(X^2) - \mu^2$	**X = result of one roll with a dice.**

1. If X is **hypergeometrically** distributed (draws without replacement) with success rate $p = R/N$, then X is for large populations **approximately $B(n,p)$**. <u>Rule of thumb</u>: $N > 5n^2$
2. If X is **$B(n,p)$-**distributed with large n and small p, then X is **approximately Poisson** distributed with **μ = np**. <u>Rule of thumb</u>: **np < 10 or $n(1-p)$ < 10.**