<u>Homework Assignment 1</u> Probability Theory 20116/2017- Hand in your own (hand written) solutions
on one A4-sheet (no e-mail) before the start of the lecture on Monday 8/5, 13.45

A guest lecturer in the Gambia wants to distribute 5 different books on statistics amongst his students, 7 female and 9 male, by drawing lots at random from a vase with the 16 names of the students. Every student can get at most one of the books. Add a brief reasoning to your calculations.

a. Find the probability that none of the female students gets a book.
b. Find the probability that at least one male and at least one female student get a book.
c. Find the probability that the first and the last book go to a male student.
d. Find the probability that the third book goes to a female student.

A small brewery has two machines, A and B, to bottle the beer.
Machine A fills 75% of the produced bottles and machine B 25%. During the quality control 1 out of 400 bottles, produced by machine A, is rejected, and of the bottles, produced by machine B, 1 out of 100 is rejected. Answer the following questions: first define relevant events, express the given (conditional) probabilities in the events and mention explicitly the properties you are applying (see for the correct notation e.g. examples 3.2.1 and 4 or solutions of exercises).

e. What proportion of all filled bottles is rejected?
f. A student buys a bottle of the beer: what is the probability that this bottle is filled by machine A?
Of course, you have to assume that the bottle has not been rejected at the quality control.

a	**b**	**c**	**d**	**e**	**f**	**Total**
1	**2**	**2**	**1**	**3**	**1**	**10**

Solutions:

	female	male	Total
Population	7	9	16
	↓	↓	↓
Sample	0	5	5

a. Evidently each student has a lot and the lots are drawn randomly and **without replacement**. For the first question the order of the draws is not important, so we can count the number of **combinations** of 5 persons. $P(0 \text{ female winners}) = \frac{\binom{7}{0}\binom{9}{5}}{\binom{16}{5}} = \frac{126}{4368} = 2.9\%$

<u>Note</u>: *It is possible to consider permutations (ordered outcomes) in this case. Usually it is advisable to count the numbers of combinations, if possible. Clearly in c you have to count permutations.*

b. Apply the Complement rule:

$$P(\text{at least one male and one female}) = 1 - [P(0 \text{ females}) + P(0\ males)]$$

$$= 1 - \left[\frac{\binom{9}{5}}{\binom{16}{5}} + \frac{\binom{7}{5}}{\binom{16}{5}}\right] = 1 - \frac{126+21}{4368} \approx 96.6\%$$

c. Now the order of the draws is essential, so compute permutations: for the number of permutations in the event choose a male on the first and last position and then fill in the other 3 positions:

$P(\text{First and fifth book go to males}) = \frac{9\times8\times14\times13\times12}{16\times15\times14\times13\times12} = \frac{3}{10} = 30\%$

<u>Note</u>: *Most students used combinations and counted $\binom{1}{1}$ as the number of possibilities to choose a boy at position 1 (should be $\binom{9}{1}$), and also other positions should be filled adequately: better choose permutations*

d. $\frac{7}{16} \approx 43.8\%$ (if we order all 16 names the probability of a male in any position is 7/16).

Note: *a direct answer as given above is allowed, using permutations of 3 out of 16 we find the same:*

$$\frac{7\times15\times14}{16\times15\times14} \approx 43.8\%$$

e. For an arbitrary bottle we define: $A =$ "filled by machine A" , so $\overline{A} =$ "filled by machine B"

$R =$ "bottle is rejected during the QC"

Given probabilities: $P(A) = 0.75$, $P(\overline{A}) = 0.25$, $P(R|A) = \frac{1}{400}$ and $P(R|\overline{A}) = 1/100$

So $P(R) = P(R|A)P(A) + P(R|\overline{A})P(\overline{A})$ (law of total probability)

$$= \frac{1}{400} \cdot 0.75 + \frac{1}{100} \cdot 0.25 = 0.004375$$

Note: *You loose 0.5 point if you did not define the events (properly), 1 point if you did not state the given probabilities correctly and 1 point for not giving the formula (law of total probability).*

f. $P(A|\overline{R}) = \frac{P(A\cap\overline{R})}{P(\overline{R})} = \frac{P(\overline{R}|A)P(A)}{P(\overline{R})}$ since $P(A) = 0.75$, $P(\overline{R}|A) = 1 - P(R|A) = \frac{399}{400}$ and

$P(\overline{R}) = 1 - P(R) = 0.995625$, we have: $P(A|\overline{R}) = \frac{\frac{399}{400}\times\cdot 0.75}{0.995625} \approx 0.7514$.

Note: *Only the correct calculation (without formula): ½ point.*