Homework assignment 2 Calculus I

a. If $f(x) = \frac{x^2-9x+14}{4x-2x^2}$, evaluate the limit if it exists :

1. [*direct substitution*] $\lim_{x\to 1} f(x) = \frac{1-9+14}{4-2} = 3$

2. $[form\ \frac{0}{0}\ , use\ factoring]$ $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x-2)(x-7)}{2x(2-x)} = \lim_{x\to 2} \frac{x-7}{2x\times(-1)} = \frac{2-7}{-2\times 2} = \frac{5}{4}$

Note: we can cancel $x-2$ since in the denominator $2-x = -1\times(x-2)$

3. $\left[form\ \frac{\infty}{\infty}, divide\ by\ x^2\right]$ $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1-\frac{9}{x}+\frac{14}{x^2}}{\frac{4}{x}-2} = \frac{1-\lim_{x\to-\infty}\frac{9}{x}+\lim_{x\to-\infty}\frac{14}{x^2}}{\lim_{x\to-\infty}\frac{4}{x}\ -2} = \frac{1-0+0}{0-2} = -\frac{1}{2}$

4. $\left[form\ \frac{c\neq 0}{0}\ , use\ factoring\ and\ reasoning\right]$ $\lim_{x\downarrow 0} f(x) = \lim_{x\downarrow 0} \frac{(x-2)(x-7)}{2x\cdot-(x-2)} = \lim_{x\downarrow 0} \frac{x-7}{-2x} = \infty$

(if $x\downarrow 0$, then x is small positive and $x-7$ is negative, so $\frac{x-7}{-2x}$ will become large positive.)

b. How would you define $f(2)$ in a. to make f continuous at 2?

Answer: define $f(2) = \frac{5}{4}$, the limiting value in exc. a.2. *For other values f remains as defined.*

c. Evaluate the limit if it exists: **1**. $\lim_{x\to 4} \frac{x^2-16}{\sqrt{x}-2}$ $\left[form\ \frac{0}{0}, use\ root\ trick\right]$

$$\lim_{x\to 4} \frac{x^2-16}{\sqrt{x}-2} = \lim_{x\to 4} \frac{x^2-16}{\sqrt{x}-2}\times\frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{(x-4)(x+4)(\sqrt{x}+2)}{x-4} = \lim_{x\to 4}(x+4)(\sqrt{x}+2) = 32$$

2. $\text{Lim}_{x\to\infty} \left(\frac{1}{4}\right)^{-x} = \lim_{x\to\infty} (4^{-1})^{-x} = \lim_{x\to\infty} 4^x = \infty$

3. [*direct substitution*] $\lim_{x\to 0} \frac{(7+x)^{-1}-7^{-1}}{7} = \frac{\frac{1}{7}-\frac{1}{7}}{7} = \frac{0}{7} = 0$

4. $\left[form\ \frac{0}{0}, use\ common\ (math)\ sense\right]$

$$\lim_{x\to 0} \frac{(7+x)^{-1}-7^{-1}}{x} = \lim_{x\to 0} \frac{\frac{1}{(7+x)}-\frac{1}{7}}{x}\times\frac{(7+x)}{(7+x)7} = \lim_{x\to 0} \frac{7-(7+x)}{7x(7+x)} = \lim_{x\to 0} \frac{-x}{7x(7+x)} = \lim_{x\to 0} \frac{-1}{7(7+x)} = -\frac{1}{49}$$

d. If $F(x) = \frac{x^2-4x}{|x-4|}$, evaluate the following limits if they exist:

Use $|x-4| = \begin{cases} x-4, & \text{if } x \geq 4 \\ -(x-4) & \text{if } x < 4 \end{cases}$, so $F(x) = \begin{cases} \frac{x^2-4x}{x-4} & \text{if } x \geq 4 \\ \frac{x^2-4x}{-(x-4)} & \text{if } x < 4 \end{cases}$

1. $\lim_{x\downarrow 4} F(x) = \lim_{x\downarrow 4} \frac{x^2-4x}{x-4} = \lim_{x\downarrow 4} \frac{x(x-4)}{x-4} = \lim_{x\downarrow 4} x = 4$

2. $\lim_{x\uparrow 4} F(x) == \lim_{x\uparrow 4} \frac{x^2-4x}{-(x-4)} = \lim_{x\uparrow 4} \frac{x(x-4)}{-(x-4)} = \lim_{x\uparrow 4}(-x) = -4$

3. $\lim_{x\to 4} F(x)$ d.n.e. , since $\lim_{x\downarrow 4} F(x) \neq \lim_{x\uparrow 4} F(x)$

e. Find the vertical and horizontal asymptotes of the function $f(x) = \frac{x^2-10x+25}{2x^2-4x-30}$.

Vertical asymptotes: if $f(x) = \frac{(x-5)^2}{2(x-5)(x+3)}$ has shape $\frac{c\neq 0}{0}$: only if $x=-3$: VA $x=-3$

HA: $y = \frac{1}{2}$, since $\lim_{x\to\infty} \frac{x^2-10x+25}{2x^2-4x-30} = \lim_{x\to\infty} \frac{1-\frac{10}{x}+\frac{25}{x^2}}{2-\frac{4}{x}-\frac{30}{x^2}} = \frac{1-0+0}{2-0-0} = \frac{1}{2}$ and

$$\lim_{x\to-\infty} \frac{x^2-10x+25}{2x^2-4x-30} = \frac{1}{2}$$