

## Homework assignment 1 Calculus I – Basic Math and Functions

Use this sheet to answer: Give a, b, c or d as an answer to MC-items 1-12 and give motivated answers to questions 13 and 14 on the other side.

		a	b	c	d	Answer
1.	$\frac{3^{-2}}{2^{-3}} =$	1	$\frac{8}{9}$	$\frac{9}{8}$	$\frac{1}{72}$	<b>b</b>
2.	Simplify $\frac{25}{5^{\frac{5}{\sqrt{5}}}}$	$5^{\frac{4}{5}}$	$5^{\frac{5}{3}}$	5	$5^{\frac{1}{5}}$	<b>a</b>
3.	$(x+6)^2 - (x+6)(x-6) =$	$12x+72$	$12x$	72	$2x^2-72$	<b>a</b>
4.	$\frac{x^2-5x}{x^2-7x+10} =$	$\frac{-5x}{-7x+10}$	$\frac{5}{x-2}$	$\frac{x^2}{x^2-2x+10}$	$\frac{x}{x-2}$	<b>d</b>
5.	If $y = \sqrt{4-x^2}$ , then the interval of permitted values of $x$ are:	$[-2, 2]$	$(-2, 2)$	$(-\infty, 2]$	$(-\infty, 2)$	<b>a</b>
6.	$\frac{x}{x+\sqrt{3}} + \frac{x}{x-\sqrt{3}} =$	$\frac{2\sqrt{3}}{x^2-3}$	$\frac{2\sqrt{3}}{x^2-2x\sqrt{3}+3}$	$\frac{2x^2}{x^2-3}$	$\frac{2x^2}{x^2-2x\sqrt{3}+3}$	<b>c</b>
7.	$y = x^2 - 10x + 16$ The number of $x$ -intercepts ( $y=0$ ) is	0	1	2	3	<b>c</b>
8.	Solve: $x^2 - 12 > x$	$x > 4$	$x < -3$ or $x > 4$	$-3 \leq x \leq 4$	$x > 4$ and $x < 3$	<b>b</b>
9.	Solve $\sin(x) = \frac{1}{2}$ for $0 \leq x \leq \pi$	$\frac{1}{6}\pi, \frac{5}{6}\pi$	$\frac{1}{6}\pi$	$\frac{1}{3}\pi$	$\frac{1}{3}\pi, \frac{2}{3}\pi$	<b>a</b>
10.	Solve $ 2x  < 3$	$-3 < x < 3$	$x < 1.5$	$-6 < x < 6$	$-1.5 < x < 1.5$	<b>d</b>
11.	If in a right angle triangle the hypotenuse is 10 and the angle is $45^\circ$ , then the adjacent side is	20	$10\sqrt{2}$	$5\sqrt{2}$	5	<b>c</b>
12.	If $\sin(x) = \frac{5}{13}$ then, for $0 < x < \frac{1}{2}\pi$ , $\cos(x) =$	0	$\frac{12}{13}$	$\frac{5}{13}$	1	<b>b</b>
13.	Assume that $f(x) = \frac{2x+3}{x-4}$ is one-to-one and find a. the inverse function $f^{-1}$ b. the domain and the range of $f$	<p>a. Show your solution in steps:</p> $y = \frac{2x+3}{x-4} \Leftrightarrow y(x-4) = 2x+3 \Leftrightarrow yx-4y = 2x+3$ $\Leftrightarrow yx-2x = 3+4y \Leftrightarrow (y-2)x = 3+4y \Leftrightarrow x = \frac{3+4y}{y-2}$ <p>Interchanging <math>x</math> and <math>y</math> we find: <math>y = f^{-1}(x) = \frac{3+4x}{x-2}</math>,</p> <p>b. <math>D_f = \{x \in \mathbb{R}   x \neq 4\}</math>, since the denominator of <math>f</math> is <math>x-4</math>.  <math>R_f = D_{f^{-1}} = \{x \in \mathbb{R}   x \neq 2\}</math> since the denominator of <math>f^{-1}</math> is <math>x-4</math>.</p>				
14.	$f(x) = x^2 - 6x = (x-3)^2 - 9$ and $g(x) = 2\sqrt{x}$ a. Is $f$ odd or even? b. Which transformation(s) on the graph $y = x^2$ should be conducted to find the graph of $f$ . c. Which transformation(s) on the graph $y = \sqrt{x}$ should be conducted to find the graph of $f$ . d. Find both $f \circ g$ and $g \circ f$ .	<p>a. <math>f</math> is neither odd nor even, since <math>f(-x) = (-x)^2 - 6 \cdot (-x) = x^2 + 6x</math></p> <p>b. First shift <math>y = x^2</math> three units to the right and then shift the resulting graph 9 units downward to obtain the graph of <math>f</math>.</p> <p>c. Stretch <math>y = \sqrt{x}</math> vertically by a factor 2.</p> <p>d. <math>f \circ g(x) = f[g(x)] = (2\sqrt{x})^2 - 6 \cdot 2\sqrt{x} = 4x - 12\sqrt{x}</math> and  <math>g \circ f(x) = g[f(x)] = 2\sqrt{x^2 - 6x}</math></p>				

Grading: Exercises 1-12 and all parts of exercise 14: ½ point

a. and b of exercise 13: each 1 point

Total 10 points

Some comments on the solutions of the MC-items:

1.	$\frac{3^{-2}}{2^{-3}} = \frac{1/9}{1/8} = \frac{1}{9} \times \frac{8}{1} = \frac{8}{9}$
2.	$\frac{25}{5^5\sqrt{5}} = \frac{5^2}{5^1 \times 5^5} = \frac{5^2}{5^{1+5}} = 5^{2-1\frac{1}{5}} = 5^{\frac{4}{5}}$
3.	$(x+6)^2 - (x+6)(x-6) = x^2 + 12x + 36 - \{x^2 - 36\} = 12x + 72$
4.	$\frac{x^2-5x}{x^2-7x+10} = \frac{x(x-5)}{(x-2)(x-5)} = \frac{x}{x-2}$
5.	$f(x) = \frac{1}{\sqrt{4-x^2}} \Rightarrow 4 - x^2 \geq 0 \Leftrightarrow x^2 \leq 4 \Leftrightarrow -2 \leq x \leq 2$ or $x$ in $[-2, 2]$
6.	$\frac{x}{x+\sqrt{3}} + \frac{x}{x-\sqrt{3}} = \frac{x(x-\sqrt{3})+x(x+\sqrt{3})}{(x+\sqrt{3})(x-\sqrt{3})} = \frac{x^2-x\sqrt{3}+x^2+x\sqrt{3}}{x^2-3} = \frac{2x^2}{x^2-3}$
7.	The number of $x$ -intercepts of $f(x) = x^2 - 10x + 16$ is the number of roots of the equation $x^2 - 10x + 16 = 0 \Leftrightarrow (x-2)(x-8) = 0 \Leftrightarrow x = 2$ or $x = 8$ : two $x$ - intercepts
8.	$x^2 - 12 > x \Leftrightarrow x^2 - 12 - x > 0 \Leftrightarrow (x-4)(x+3) > 0$ This is the case if both factors, $(x-4)$ and $(x+3)$ , are positive or both are negative $\Rightarrow x < -3$ or $x > 4$
9.	$\sin(x) = \frac{1}{2} \Leftrightarrow x = \frac{1}{6}\pi$ or $x = \frac{5}{6}\pi$ (These are the only solutions within $[0, \pi]$ )
10.	$ 2x  < 3 \Leftrightarrow -3 < 2x < 3 \Leftrightarrow -1.5 < x < 1.5$
11.	$\cos(45^\circ) = \frac{adj}{hyp} \Leftrightarrow \frac{1}{2}\sqrt{2} = \frac{adj}{10} \Leftrightarrow adjacent = 10 \times \frac{1}{2}\sqrt{2} = 5\sqrt{2}$
12.	$\sin^2(x) + \cos^2(x) = 1$ , so $\cos^2(x) = 1 - \left(\frac{5}{13}\right)^2 = \frac{144}{169}$ and $\cos(x) = \pm \frac{12}{13}$ , Since $0 < x < \frac{1}{2}\pi$ , $\cos(x) = \frac{12}{13}$