4.2,4.3.

$$\mathbf{P} = \begin{array}{c} (RRR) \\ (RRD) \\ (RDR) \\ (RDD) \\ (DRR) \\ (DRD) \\ (DDR) \\ (DDD) \end{array} \left\| \begin{array}{cccccccc} (RRR) & (RRD) & (RDR) & (RDD) & (DRR) & (DRD) & (DDR) & (DDD) \\ .8 & .2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .4 & .6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .6 & .4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .4 & .6 \\ .6 & .4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .4 & .6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .6 & .4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .2 & .8 \end{array} \right\|$$

4.10. The answer is $1 - P^3_{0,2}$ for the Markov chain with transition matrix

$$\left\| \begin{array}{ccc} .5 & .4 & .1 \\ .3 & .4 & .3 \\ 0 & 0 & 1 \end{array} \right\|$$

4.14 1) $\{0,1,2\}$ recurrent; 2) $\{0,1,2,3\}$ recurrent;
3) $\{0,2\}$ recurrent, $\{1\}$ transient, $\{3,4\}$ recurrent;
4) $\{0,1\}$ recurrent, $\{2\}$ recurrent, $\{3\}$ transient, $\{3\}$ transient.

4.15 Hint: Show that if there is a path from i to j then there always exists a path $i_0 = i, i_1, \dots, i_n = j$ such that $i_k \neq i_l$ for $k \neq l$; $k, l = 1, \dots, n$.

4.20. Show that if $\sum_{i=0}^M P_{ij} = 1$ for all $j = 0, \dots, M$ then $\pi_j = 1/(M+1)$ satisfies

$$\pi_j = \sum_{i=0}^M \pi_i P_{ij}, \quad \sum_{j=0}^M \pi_j = 1,$$

and then use the uniqueness property.

4.25. Let X_n be the number of pairs of shoes at the front door just before the runner departs on day n. Then $\{X_n, n \geq 1\}$ is a Markov chain. The transition probabilities are as follows. For $i = 1, \dots, k-1$, we have $P_{i,i} = 1/2$, $P_{i,i-1} = 1/4$, $P_{i,i+1} = 1/4$. Further, $P_{0,0} = 3/4$, $P_{0,1} = 1/4$, $P_{k,k} = 3/4$, $P_{k,k-1} = 1/4$. Note that the transition matrix is doubly-stochastic and apply 4.20.

4.29. $\pi_1 = 6/17, \pi_2 = 7/17, \pi_3 = 4/17$. According to the law of large numbers, if N is large, then approximately 6,7, and 4 of each 17 employees are in categories 1,2, and 3.

4.30. Let X_n be 0 if the nth vehicle is a car and 1 if it is a truck. Then $\{X_n, n \geq 1\}$ is a two-state Markov chain with transition probabilities

$$P_{00} = 4/5, P_{01} = 1/5, P_{10} = 3/4, P_{11} = 1/4.$$

The long-run proportions are the solution of

$$\pi_0 = 4/5\,\pi_0 + 3/4\,\pi_1, \quad \pi_1 = 1/5\,\pi_0 + 1/4\,\pi_1, \quad \pi_1 + \pi_2 = 1.$$

The result is $\pi_0 = 15/19$, $\pi_1 = 4/19$. That is, 4 out of 19 vehicles is a truck.

4.57. Let A be the event that all states are visited before time T. Then conditioning on the first step we get

$$P(A) = pP(A|\text{1st step clockwise}) + qP(A|\text{1st step counterclockwise})$$
$$= p\,\frac{1-q/p}{1-(q/p)^n} + q\,\frac{1-p/q}{1-(p/q)^n}.$$

The conditional probabilities above are obtained by noticing that they are equal to the probability in the gambler's ruin problem that a gambler that starts with 1 will reach n before going broke when the gambler's win probabilities are p and q.

4.63. $s_{1,3} = 0.621$, $s_{2,3} = 0.896$, $s_{3,3} = 1.931$, $f_{1,3} = 0.3214$, $f_{2,3} = 0.4643$, $f_{3,3} = 0.4821$.