Master's thesis

Draft report

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Modeling offshore basin as deep water and shallow water part.

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Quasi-linear formulation of the shallow water equation in one spatial dimension

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = -g\frac{\partial b}{\partial x}, \tag{1}$$

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0, \tag{2}$$

Dimensionless form of the quasi-linear formulation (1)-(2) in a form, conservative for b,

$$\frac{\partial u}{\partial t} + u\frac{\partial f(u)}{\partial x} = S \tag{3}$$

with $u = (hu, h)^T$ and $S = (-gh\frac{\partial b}{\partial x}, 0)^T$, topographic term S, and transpose $(., .)^T$

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Finite volume scheme

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Riemann problem

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Results

3 Potential flow model

3.1 Introduction

The model of the offshore basin at MARIN considered in this report consists of a shallow water part and a deep water part. In the previous chapter the shallow water part was treated. In this chapter a potential flow model for the deep is presented. This chapter begins with a description of the mathematical model and gives analytical solutions for some cases. Subsequently the numerical scheme for the potential flow model is presented. This chapter ends with a comparison between the numerical solutions and the analytical solutions

3.2 Mathematical model

The waves in the deep water part of the offshore can be described by a linear potential flow model, which is given by Laplace's equation

$$-\nabla^2 \phi = 0 \text{ on } \Omega, \tag{4}$$

and boundary conditions at the free surface $\partial\Omega_S$

$$\partial_t \phi + g\eta = 0, \text{ and} \tag{5}$$

$$\partial_t \eta - \partial_z \phi = 0, \tag{6}$$

and a no normal flow boundary condition on the rigid bed $\partial\Omega_b$:

$$\mathbf{n} \cdot \nabla \phi = 0 \tag{7}$$

The wave maker is located at the left boundary, which involves a boundary condition with a prescribed normal velocity. The modeling of the wave maker is explained in detail in the next paragraph.
Depending on the application of interest, the boundary condition at the right of the domain can be a Neumann (fixed wall) or periodic boundary condition.
domain
assumptions: irrotional, incompressible, inviscid flow

3.3 Modeling of the wave maker

How to model the wave maker.

3.4 Analytical solutions

For the potential flow model (4)-(7) analytical solutions can be obtained. Suppose the solution is of the form

$$\phi(x, z, t) = \hat{\phi}(x, z)e^{iwt} \tag{8}$$

Then because of (4) we have on Ω:

$$\nabla^2(\hat{\phi}(x, z)e^{iwt}) = 0,$$

or

$$\hat{\phi}_{xx}e^{iwt} + \hat{\phi}_{zz}e^{iwt} = 0,$$

so

$$\nabla^2\hat{\phi}(x, z) = 0. \tag{9}$$

Boundary conditions (5) and (6) on $\partial\Omega_S$ can written as a single boundary condition, by differentiating (5) with respect to t and substituting (6) into (5), resulting in:

$$\partial_(tt)\phi - \partial_z\phi = 0 \text{ on } \partial\Omega_S. \tag{10}$$

Putting (8) into the free surface boundary condition (10) gives

$$i^2\omega^2\hat{\phi}e^{iwt} + \partial_z\hat{\phi}e^{iwt} = 0 \text{ on } \partial\Omega_S,$$

or

$$e^{iwt}(\partial_z\hat{\phi} - \omega^2\hat{\phi}) = 0 \text{ on } \partial\Omega_S,$$

resulting in

$$\partial_z\hat{\phi} - \omega^2\hat{\phi} = 0 \text{ on } \partial\Omega_S, \tag{11}$$

Substuting (8) in (7) leads to:

$$\partial_z\hat{\phi}e^{i\omega t} = 0 \text{ on } \partial\Omega_b,$$

or

$$\partial_z\hat{\phi} = 0 \text{ on } \partial\Omega_b, \tag{12}$$

Now we apply the method of separation of variables for (9), (11) and (12).
Suppose

$$\hat{\phi}(x, z) = f(x)g(x),$$

Then because of (9) we have on Ω:

$$\nabla^2(\hat{\phi}(x, z) = \nabla^2(f(x)g(x)) = 0,$$

or

$$\frac{\partial^2 f}{\partial x^2}g + \frac{\partial^2 g}{\partial z^2}f = 0$$

Boundary condition (11) (at the free surface $\partial\Omega_S$) then becomes:

$$f\partial_z g - \omega^2 fg = 0 \text{ on } \partial\Omega_S,$$

so

$$f(\partial_z g - \omega^2 g) = 0 \text{ on } \partial\Omega_S.$$

Because $f \neq 0$ (trivial solutions not allowed):

$$\partial_z g - \omega^2 g = 0 \text{ on } \partial\Omega_S.$$

Deviding by fg gives:

$$\frac{\partial^2 g}{\partial z^2}\frac{1}{g} + \frac{\partial^2 f}{\partial x^2}\frac{1}{f}.$$

Because $\frac{\partial^2 g}{\partial z^2}\frac{1}{g}$ only depends on z and $\frac{\partial^2 f}{\partial x^2}\frac{1}{f}$ only on x, we have:

$$\frac{\partial^2 g}{\partial z^2}\frac{1}{g} = k^2 \text{ and } \frac{\partial^2 f}{\partial x^2}\frac{1}{f} = -k^2 \text{ , with } k \text{ a constant.}$$

Boundary condition (12) becomes:

$$f\partial_z g = 0 \text{ on } \partial\Omega_b,$$

because $f \neq 0$:

$$\partial_z g = 0 \text{ on } \partial\Omega_b. \tag{13}$$

Analytical solutions for certain boundary conditions and initial conditions, with and without wave maker.

3.5 Numerical scheme

Description of the numerical scheme used in the potential flow code.

3.6 Results

Comparison between analytical and numerical solutions for certain (test) cases.