Constructing a wave reflector to protect beaches?

Modelling assignment for
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One possible way to protect beaches from incoming waves would be to try to partially reflect the waves by constructing suitable bottom variations in the sea in front of the beach. To investigate the viability of this basic idea is the aim of this modelling assignment. The full investigation is still outside reach, even for the best specialists.

The basic assumption that we will make is that the gravity-driven surface waves are considered to be of low amplitude, which allows us to restrict to linear waves. Furthermore, assuming shallow water, dispersive effects are neglected which means that we assume that each sinusoidal wave travels with a velocity that depends only on the depth of the water. Denoting the bottom topography measured from some still-water level by h, the propagation velocity is then given by $c = \sqrt{gh}$, where g is the gravitational constant.

You could contemplate a while on this velocity-depth relation: motivate it from dimensional analysis. You could also use it to motivate the assumption below that for 'natural' beaches it is a good approximation to assume that waves coming in from the deep sea will approach the beach perpendicular.
We consider a one-dimensional model, and consider incoming waves in the direction perpendicular to the beach. We consider incoming waves with one fixed frequency (which will be an important parameter in the problem), so-called monochromatic waves, as motivated by the assumed linearity of the problem.
Derive the equation for mono-chromatic waves and conclude that even for this, severely simplified model, no explicit solutions in closed form can be found for general bottom topography. Note that for waves above constant depth you will arrive at the common expression for mono-chromatic waves.
Variations in the depth will lead to partial reflection of incoming waves. This is most extreme for sudden jumps in the depth, for instance when a wave above a horizontal bottom meets a bar in the bottom.
Investigate the reflection-transmission properties of an incoming wave of given amplitude when it meets a single bar in the bottom as shown in the picture below (assume constant bottom at the left and at the right of the bar). Derive expressions for the amplitude of the reflected and the transmitted wave. Use the fact that above constant depth the solution is known, and try to match such solutions in the three different regions by deriving suitable 'interface-conditions' at the positions of the jumps.
Study the dependence of the reflection coefficient on the parameters: frequency, height and length of the bar.
Construct the most effective wave reflector(s) for waves of given frequency.
Study the reflection properties of such an 'optimal' reflector for nearby frequencies. (Why would one be interested to study this?)
In the above investigation (with infinite regions left and right of the bar) no effect of the presence of a 'beach' is incorporated.
Derive and study a simple model when a (totally reflecting) beach is present. Discuss and conclude about viability.