Hydraulic and slurry flows through a channel contraction

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Outline

- 1. Introduction
- 2. Experiments
- 3. Multiple steady states: 1D theory
- 4. Supercritical flow: 2D theory
- 5. Slurry flows: 2D simulations
- 6. Conclusions
- References

1. Introduction

- We will consider shallow water flows through a contraction, experimentally, analytically and numerically.
- Large variations in water discharges through contracting channels can lead to dramatic changes in the flow state.
- Surges: overflow of rivers underneath bridges and in ravines.
- Linear contraction: archetypical contraction geometry.
- E.g. contraction flow with two oblique hydraulic jumps:

...Introduction

Fig. 1. Oosterschelde storm surge barrier.

. . . Introduction

Motivation:

- How do hydraulic flows through a contraction compare with granular flows through a contraction, latter on inclined chutes?
- Can we experimentally find three stable steady states for one discharge rate? Baines and Whitehead (2003) could not, for flow over a hill.

2. Experiments

- Horizontal flume: 110cm long, $b_0 = 20$cm wide, with sluice gate:

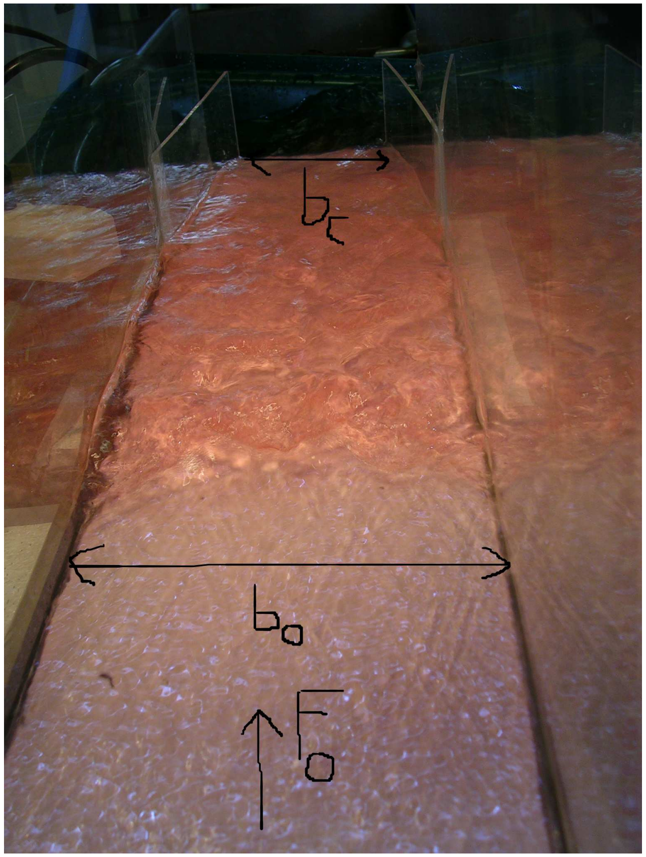


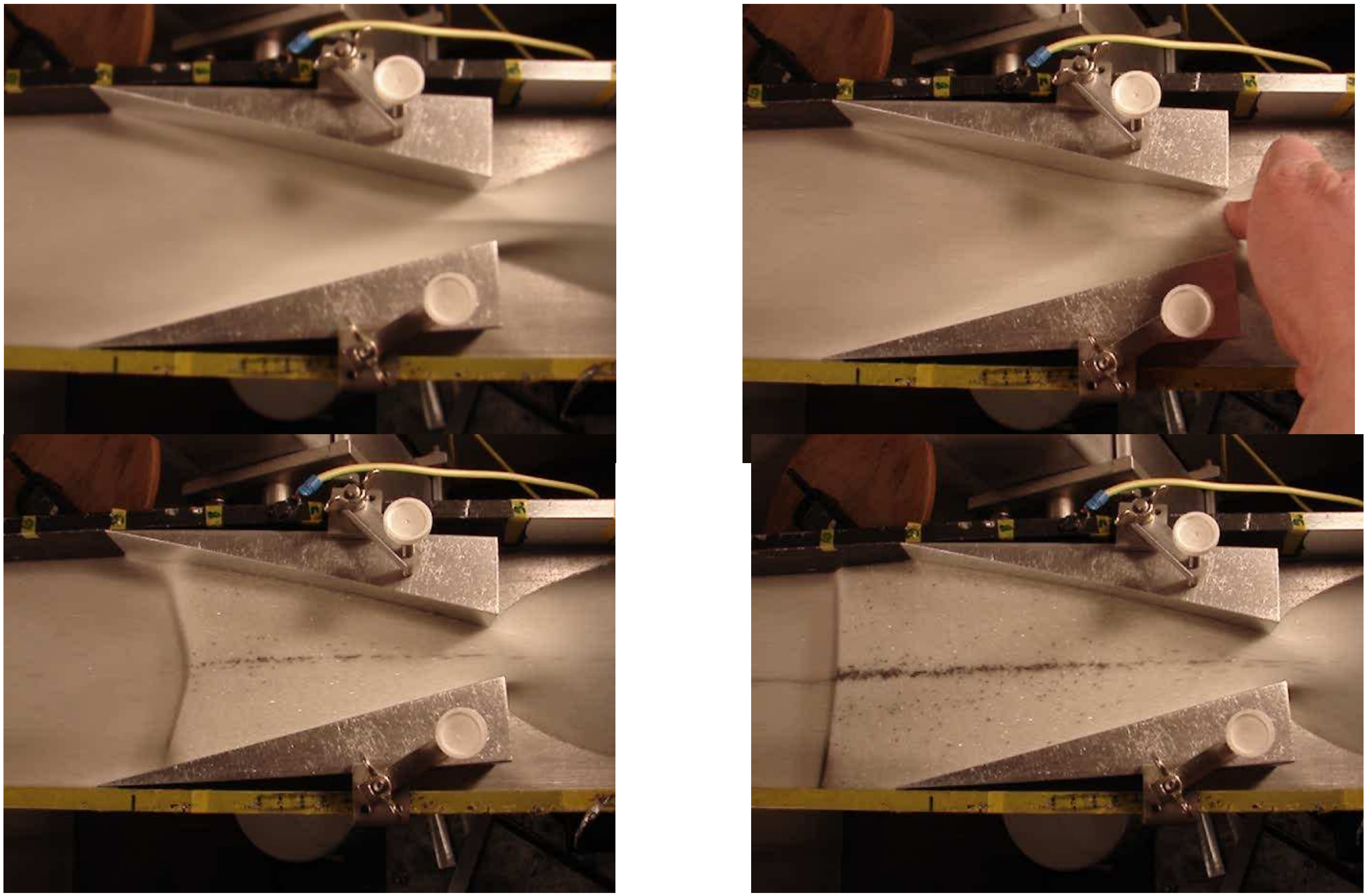
Fig. 2. Looking at the contraction in the flume.

- Three multiple steady states in regions (i/iii/iv):

Fig. 4. Multiple states: $F_0 = 3.07$ & $B_c = 0.7$. Left to right: upstream steady shock (iii), reservoir state with "Mach stem" (iv), and oblique waves (i). Transitions induced by pushing flow.

Compare with dry granular flow on inclined chute:

- Transition from supercritical to granular jump state:



- Steady-state flows: parameter plane of upstream Froude number F_0 and scaled minimum nozzle width $B_c = b_c/b_0$:

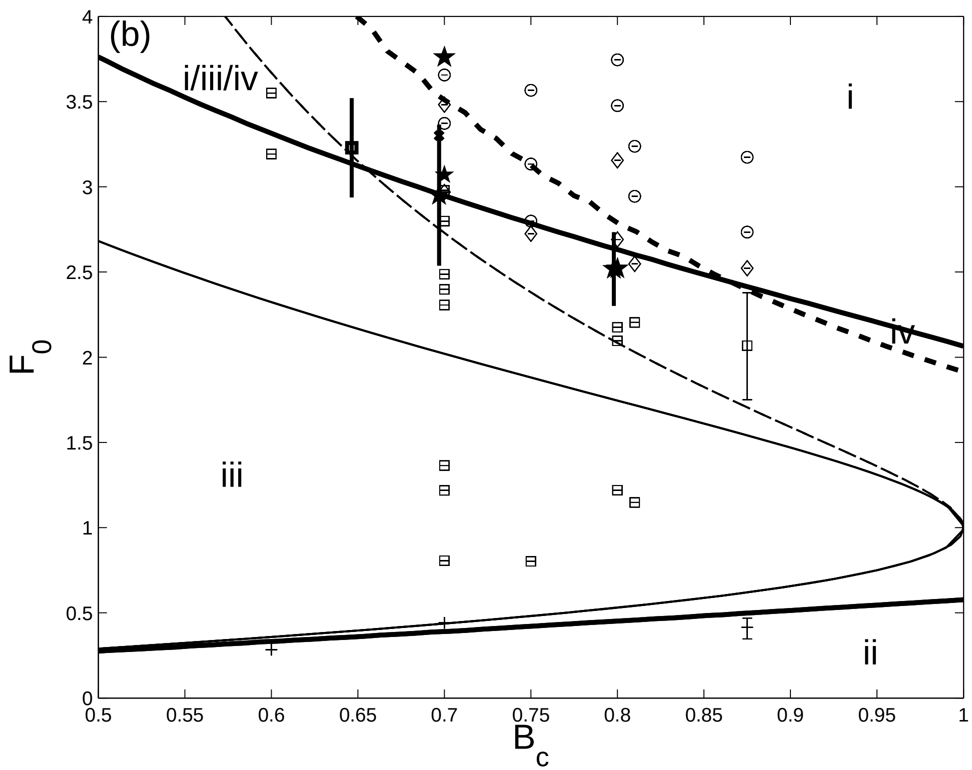


Fig. 3. +: smooth flows (ii); □: upstream moving shocks (iii); ◇: steady shocks (iii); o: oblique waves (i). ⋆: flows with 3 possible states.

- Stars, three multiple steady states in regions (i/iii/iv):

Fig. 4'. Multiple states: $F_0 = 3.07$ & $B_c = 0.7$. Left to right: upstream steady shock (iii), reservoir state with "Mach stem" (iv), and oblique waves (i). Transitions induced by pushing flow.

- Transition induced by upstream avalanche of polystyrene beads:

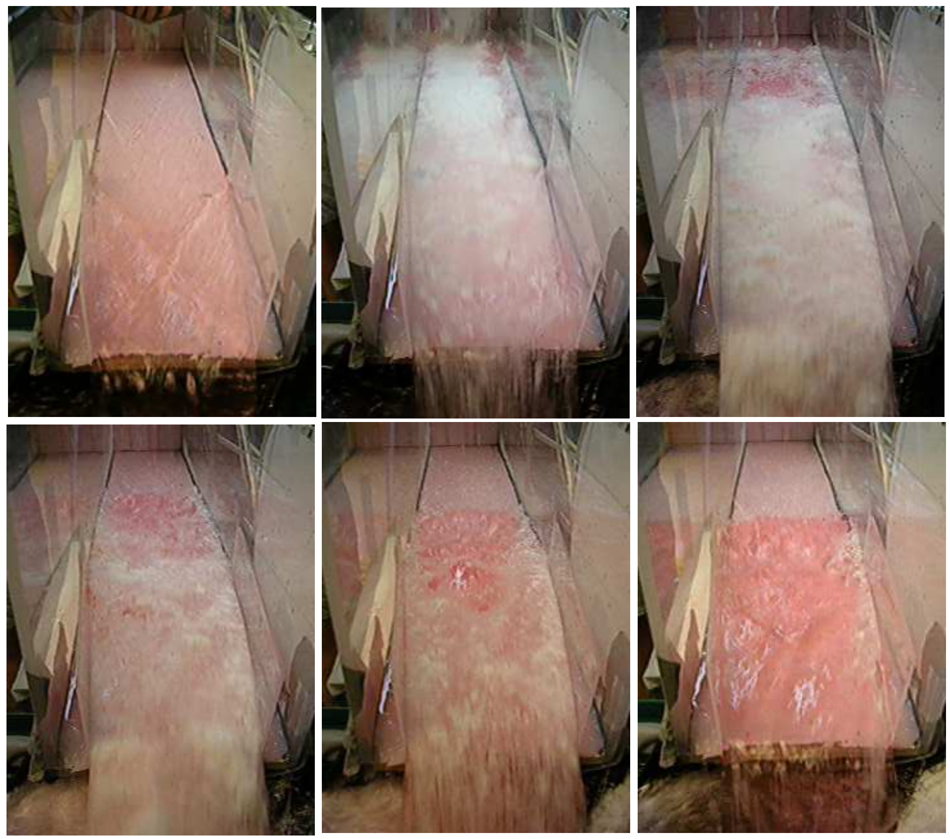


Fig. 5. Oblique wave state (top left) to upstream steady shock state (bottom right). t=1 to 6s. $\sim 900\, kg/m^3$, $F_0 = 3.07$ & $B_c = 0.7$.

3. Multiple steady states: 1D theory

- Steady-state and "shock" solutions of dimensionless 1D cross-sectionally averaged shallow water equations:

$$u_t + u\, u_x + h_x/F_l^2 \quad = \quad -C_d\, u^2/h \qquad (1)$$

$$(bh)_t + (buh)_x \quad = \quad 0 \qquad (2)$$

- with velocity $u = u(x,t)$, depth $h = h(x,t)$, width $b = b(x)$,
- Froude number $F_l = F_0$ upstream or $F_l = F_m$ at the contraction entrance, and
- quadratic friction with coefficient C_d.
- Yielding the demarcation lines in Fig. 3"' (thin: $C_d = 0$, thick: $C_d > 0$).

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

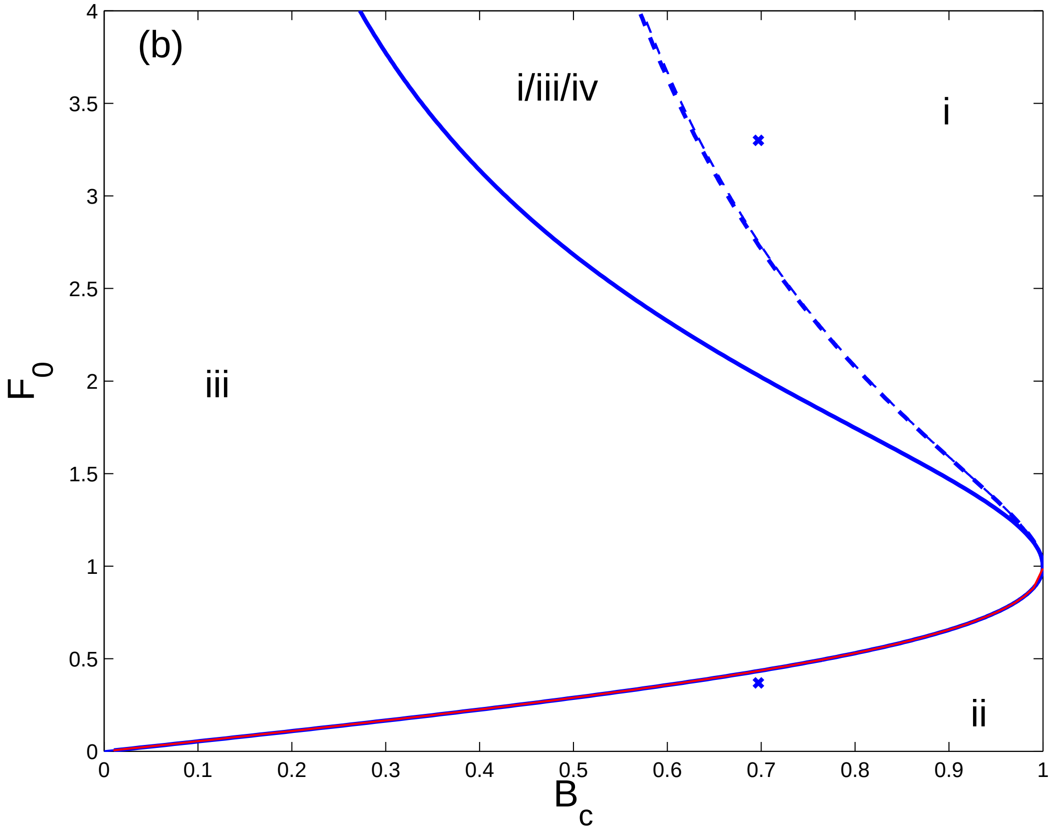


Fig. 3'. From 3 (inviscid) . . . .

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

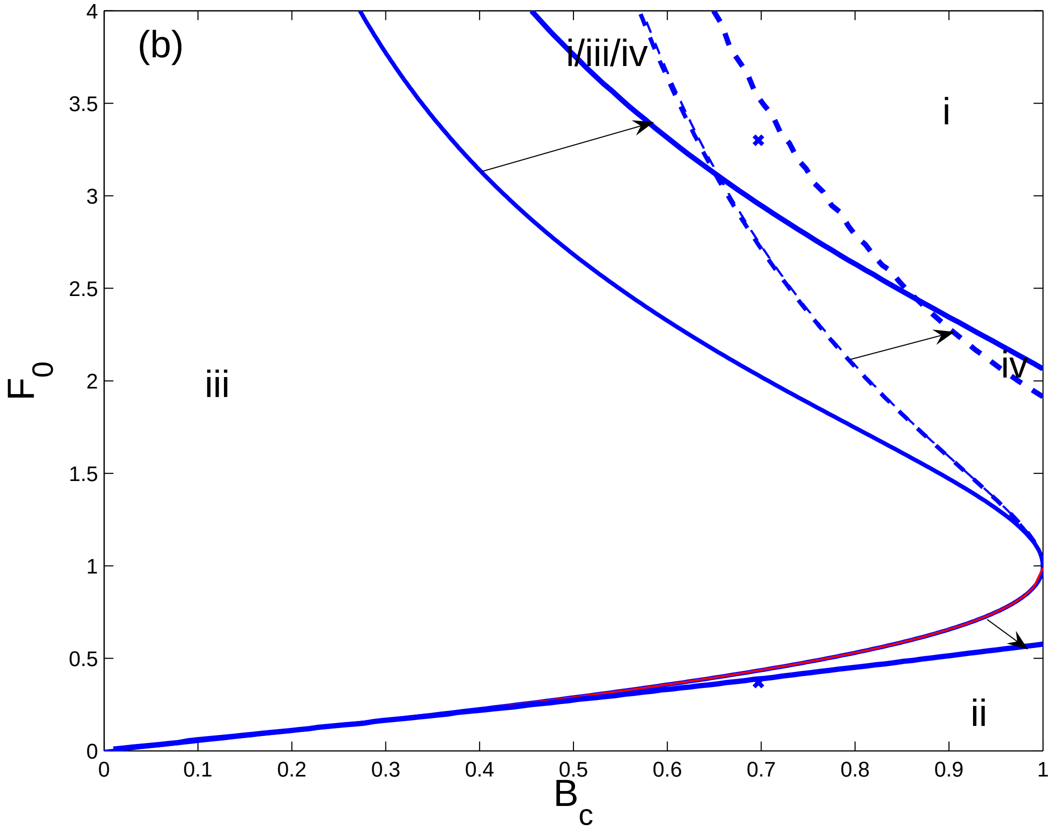
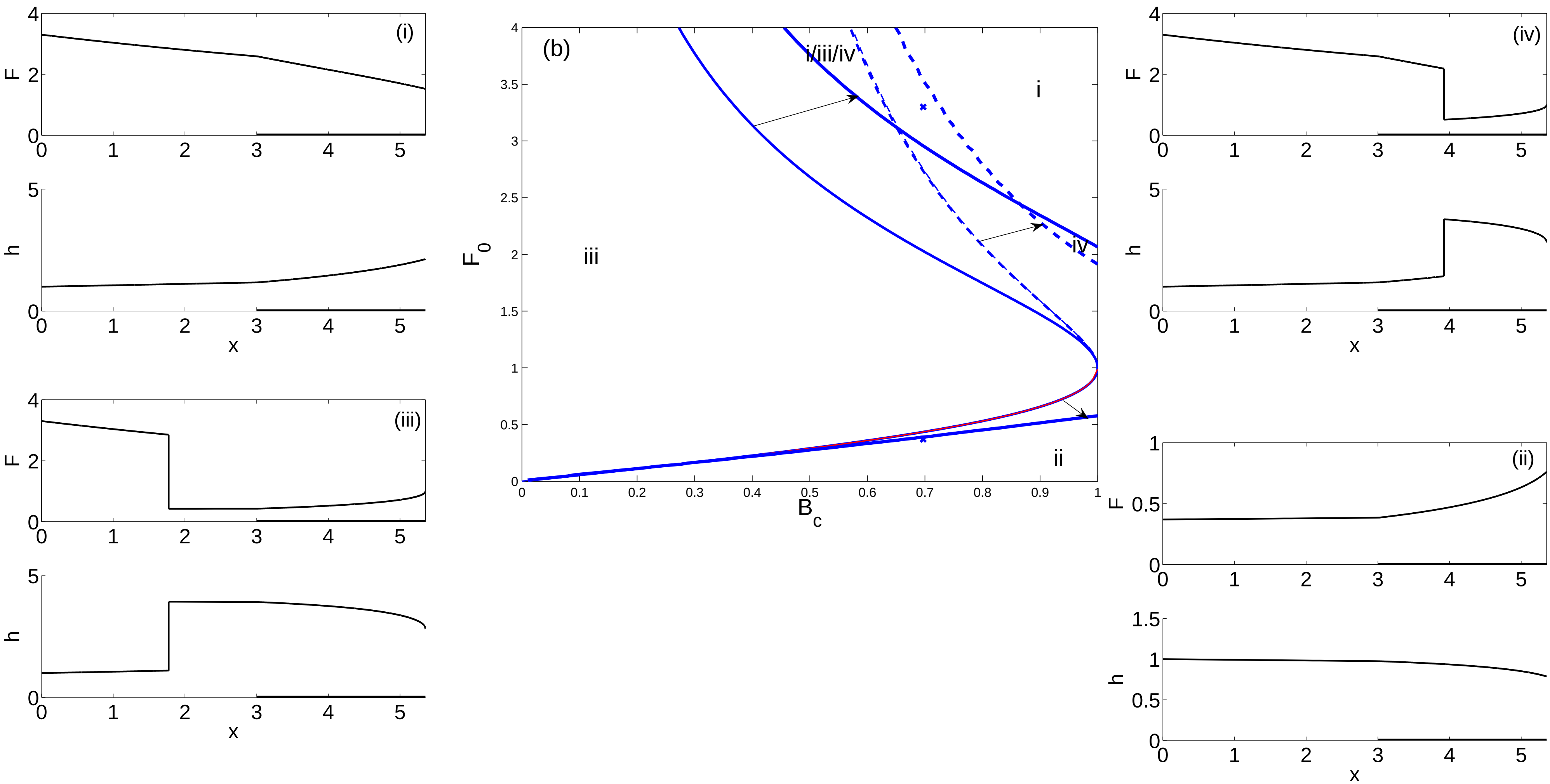
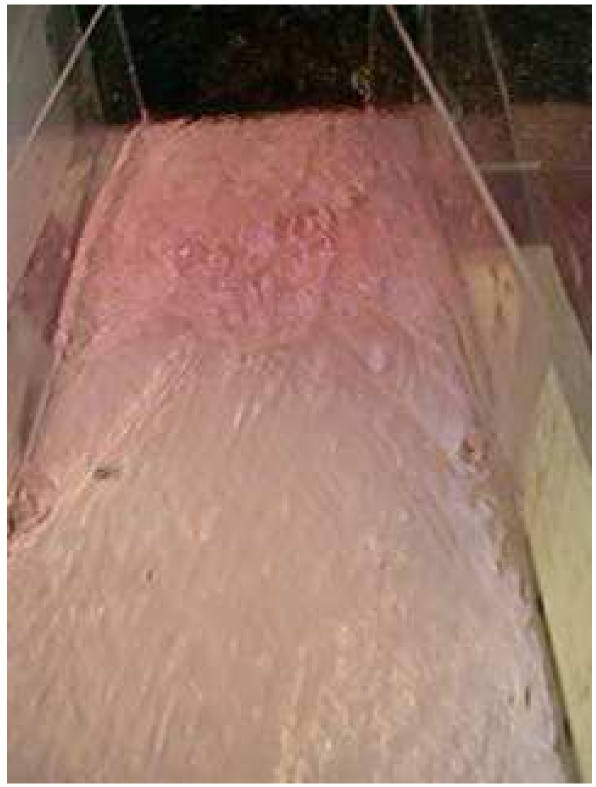


Fig. 3". . . . 3 (inviscid) to 4 (friction) flow regimes: bifurcation.

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:



- Three multiple steady states in regions (i/iii/iv):


- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

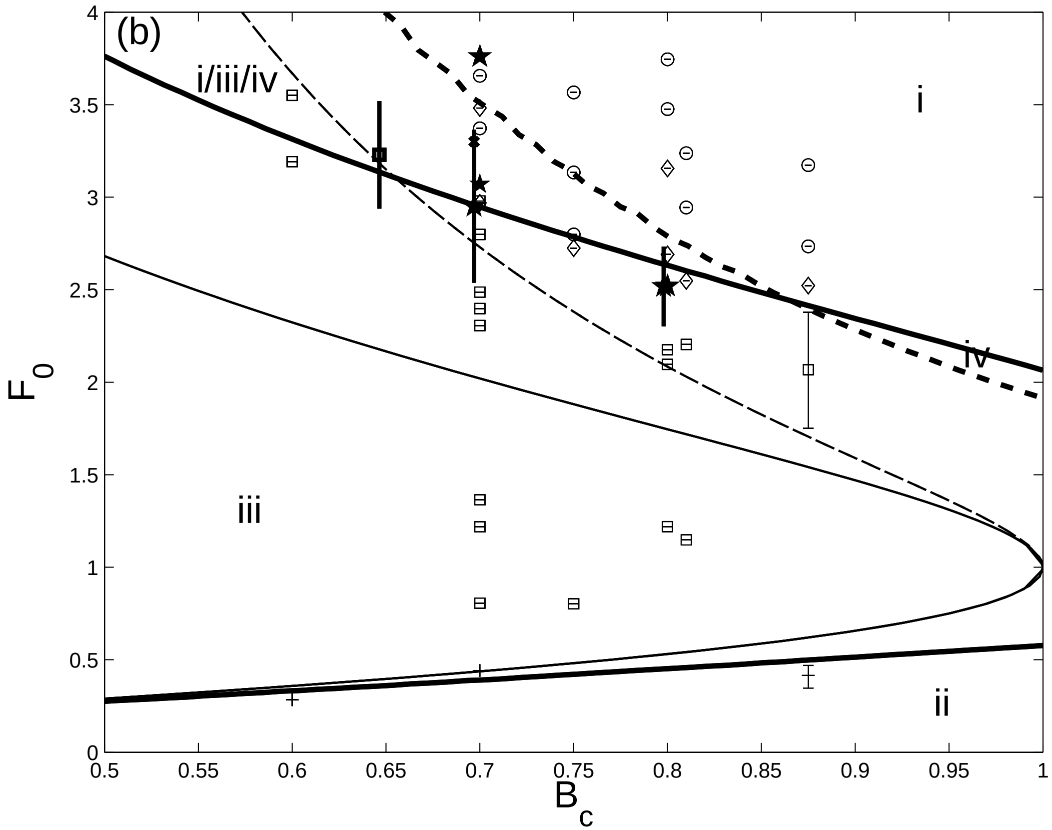


Fig. 3"'. +: smooth flows (ii); □: upstream moving shocks (iii); ◇: steady shocks (iii); ○: oblique waves (i). ⋆: flows with 3 possible states.

4. Supercritical flows: 2D theory

- Some people did not believe 1D theory.

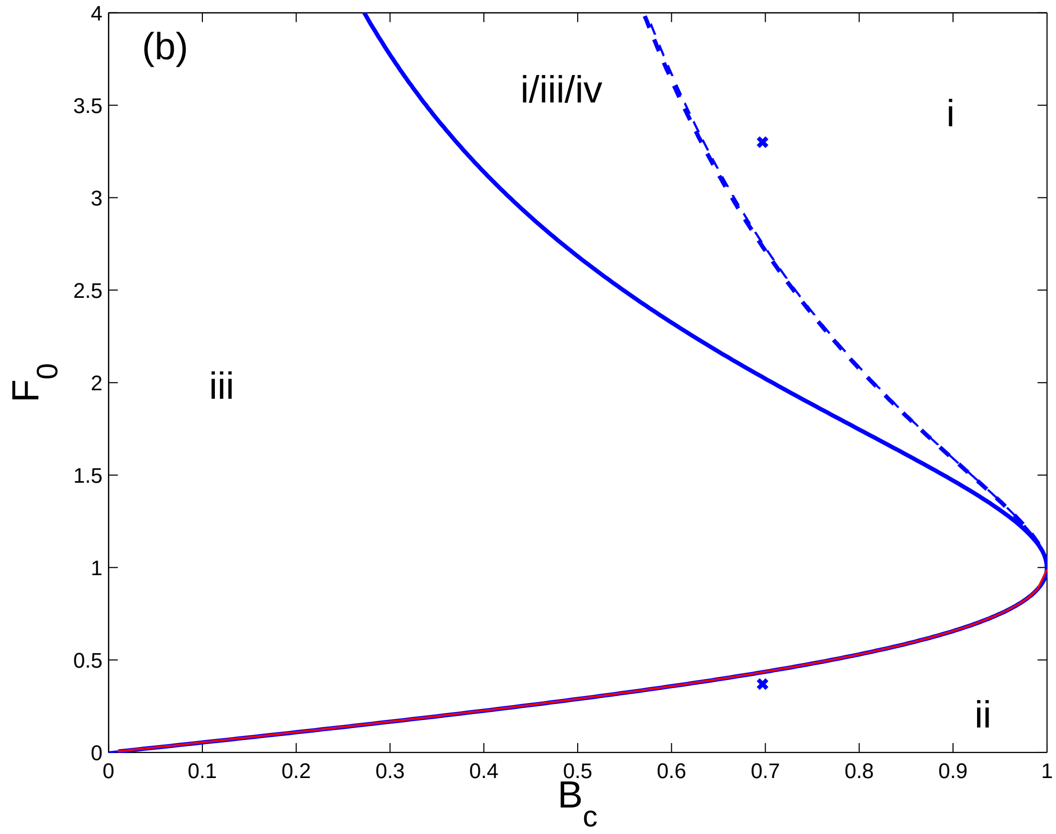


Fig. 3". . . . 3 Inviscid case.

. . . 2D theory

- Existence of 2D oblique hydraulic jumps for zero bulk friction:

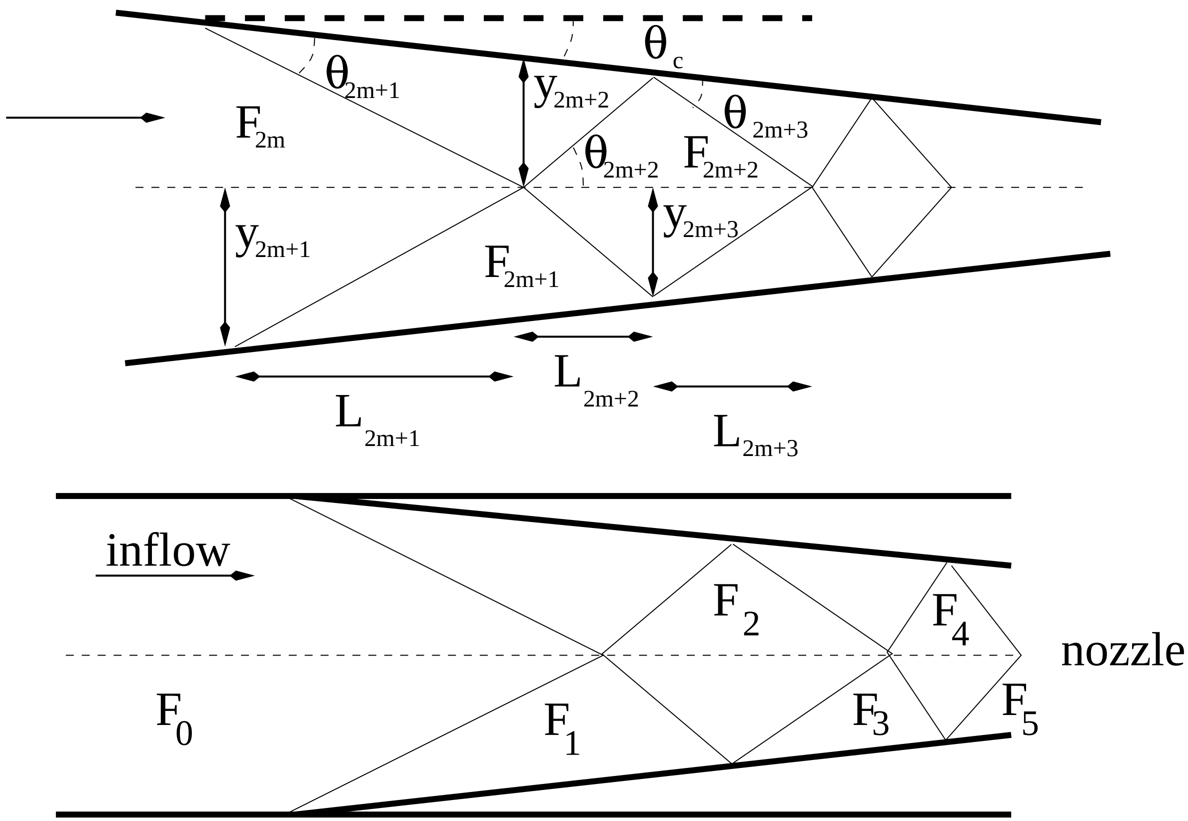
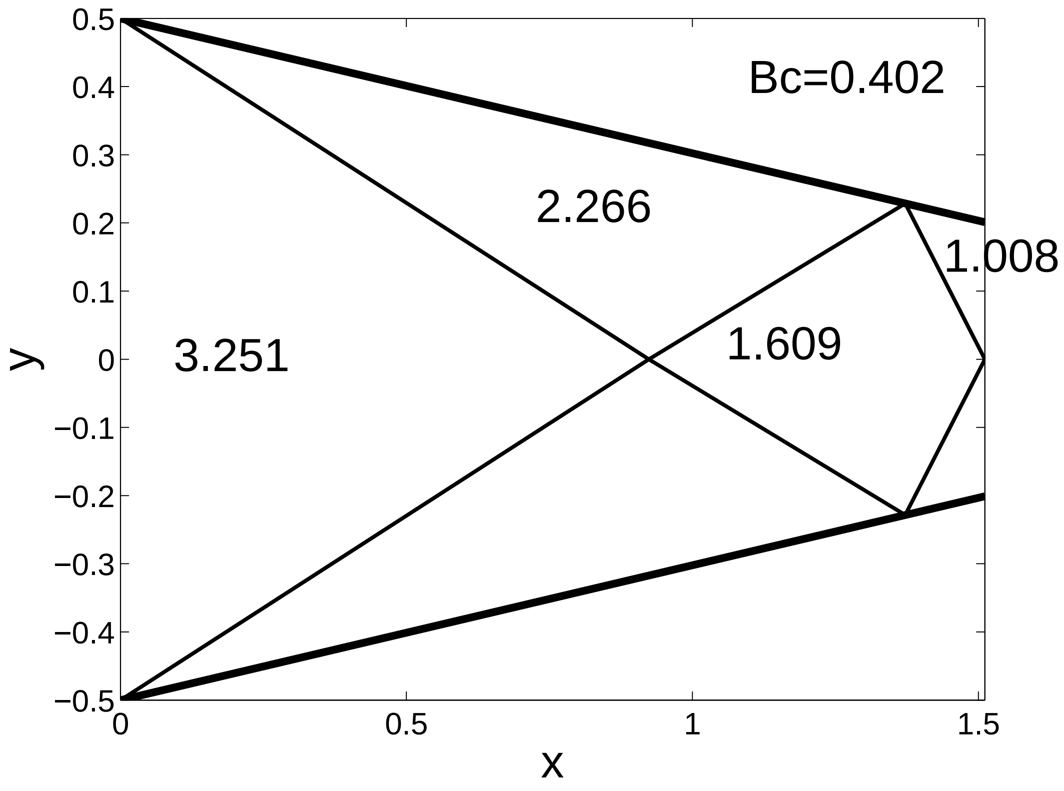


Fig. 6. Sketch of angles θ and Froude numbers F_{2m}, F_{2m+1}.

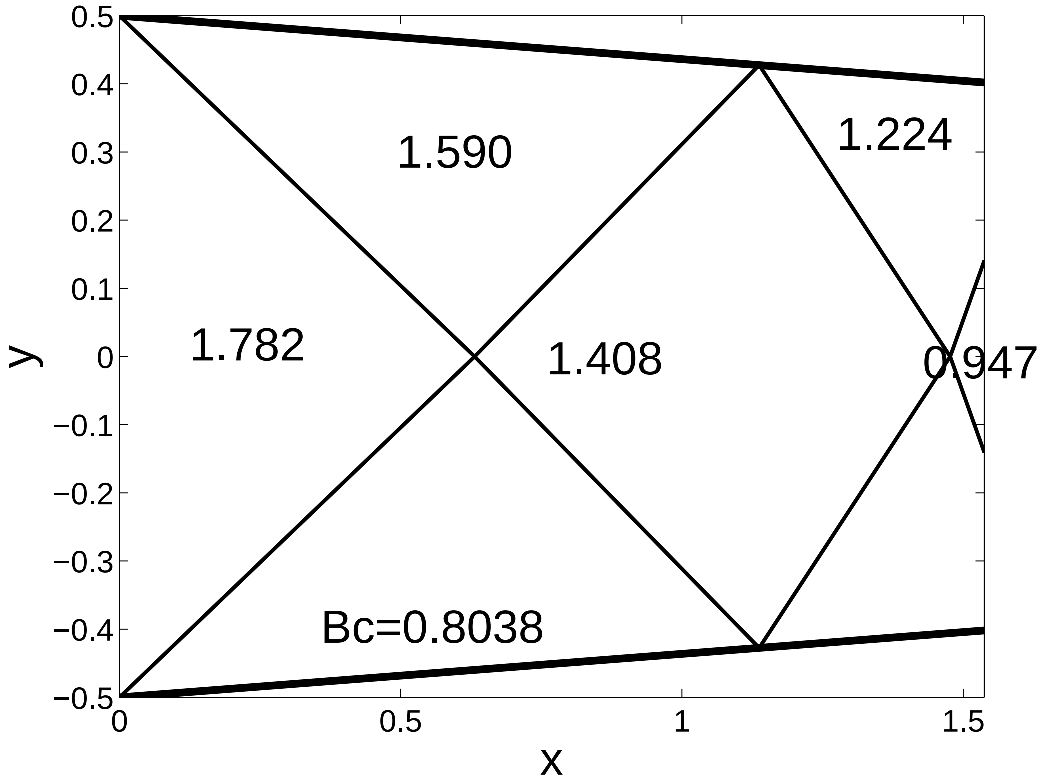
. . . 2D theory

- when the downstream angles θ_{2m+1} or θ_{2m+2} can be calculated for Froude numbers above a critical F_0, or



...2D theory

- Froude number of last polygon entirely fitting within contraction lies above unity for certain critical F_0 at inflow:



- Demarcation between supercritical (smooth) solutions and upstream moving jumps determined with 1D hydraulic theory, 2D theory for oblique hydraulic jumps, and by simulations.

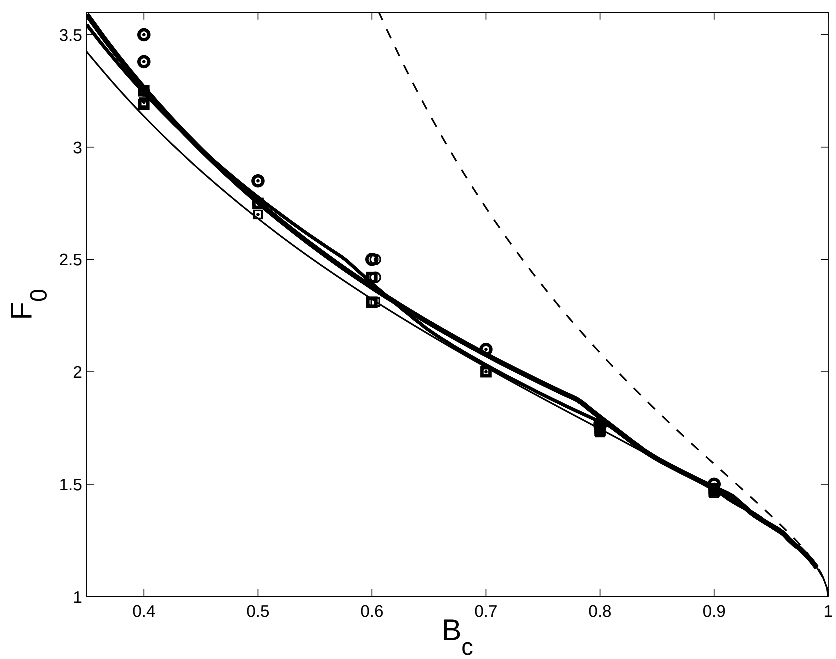
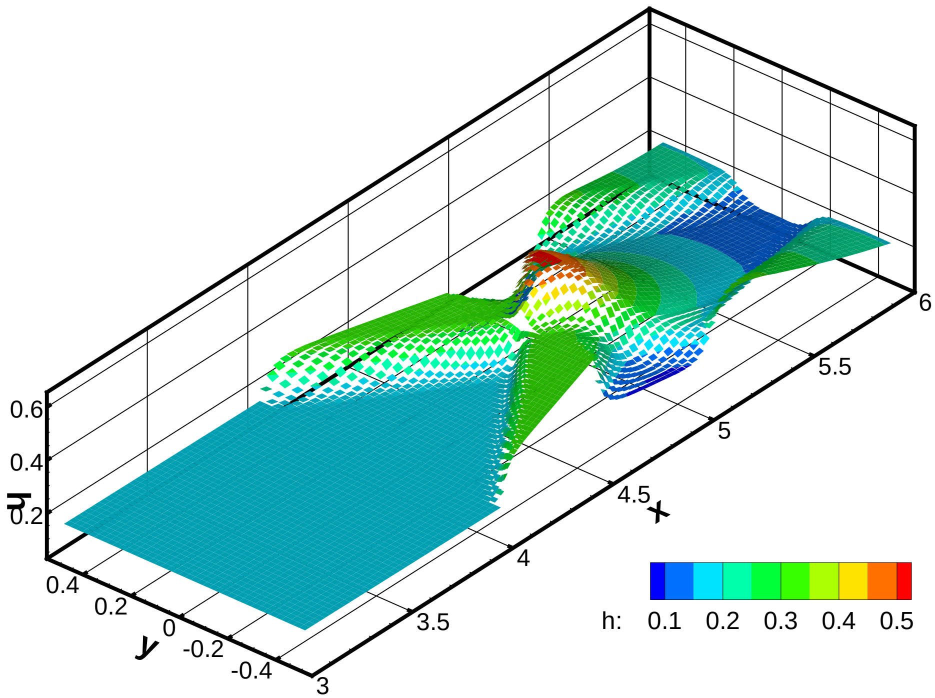
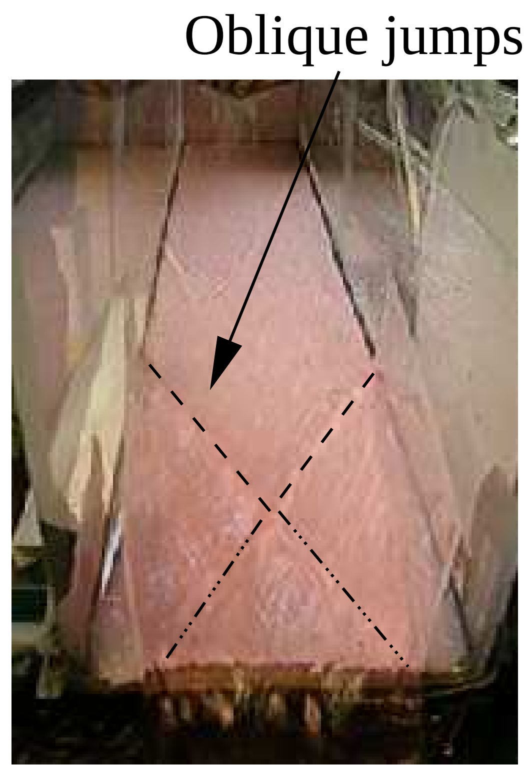


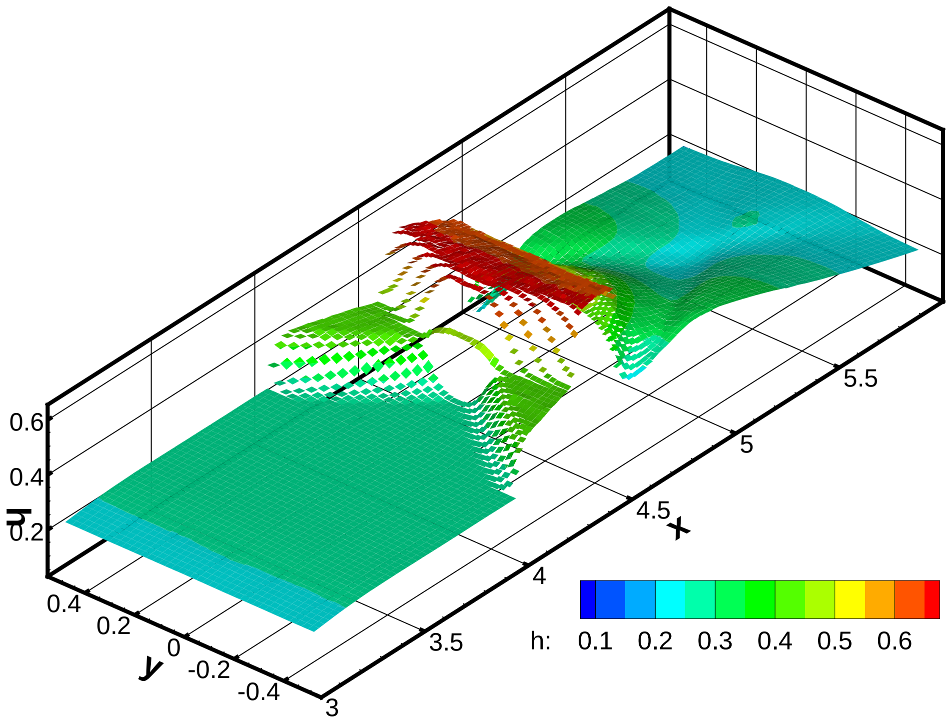
Fig. 7. 1D theory (thin solid); 2D theory: paddle lengths $L = 0.305m$ (thicker) and $L = 0.465m$ (thickest); and, simulations $L = 0.305m$ ($\circ/\square$) and $L = 0.465m$ ($\circ/\square$ with dot in center).

5. Slurry Flows

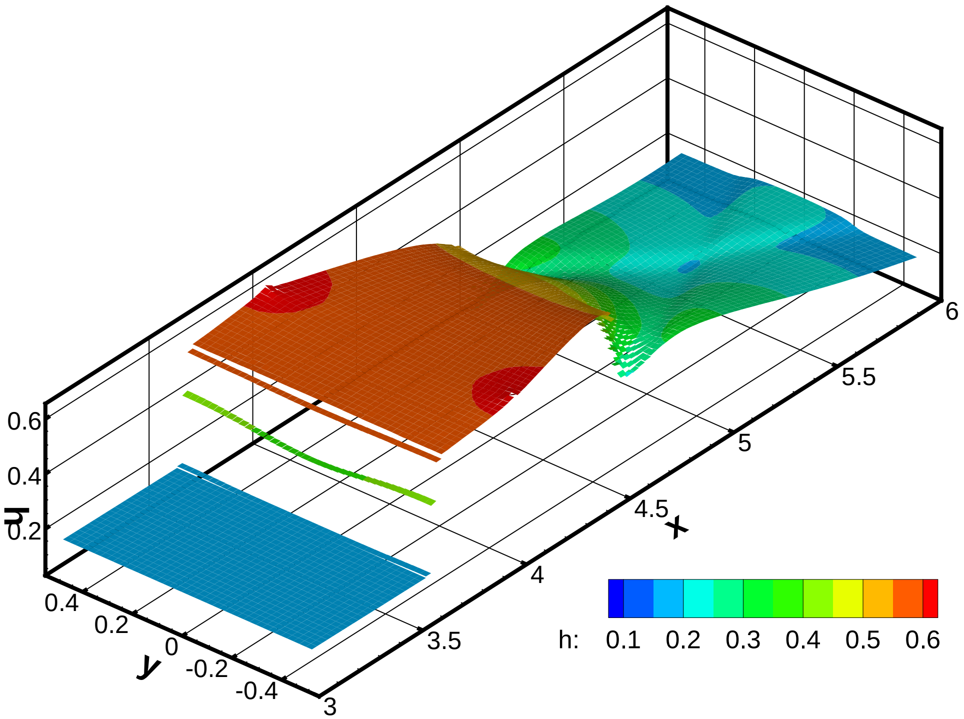


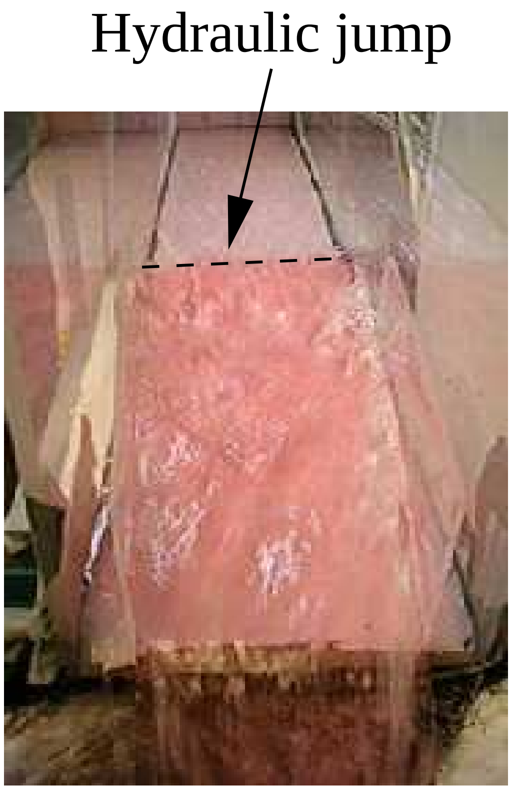


Depth-averaged 2-phase model



Discontinuous Galerkin FEM





6. Conclusions

- Experiments and 1D theory with turbulent friction match reasonably well.
- 1D theory demarcates the 2D supercritical flow boundary surprisingly well, for inviscid bulk flow.
- Similar experiments and theory done for granular chute flows (Vreman,...,B. JFM, 2007).
- Extension to 2D hydraulic and morphodynamic theory of oblique jumps in progress.
- Experiments with fluid-particle slurry flows in progress.

References

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