Dynamics of fluid-filled gelatin cracks

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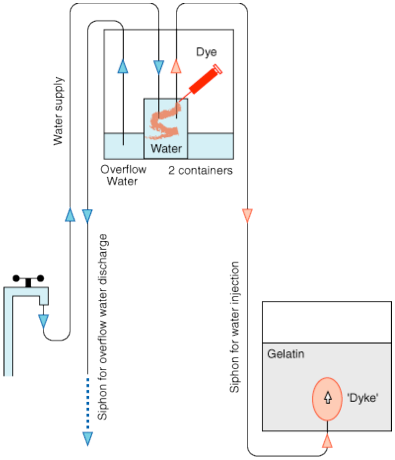
Outline

- 1. Motivation
- 2. Conceptual experimental results
- 3. Elasto-hydrodynamic lubrication model
- 4. Discussion
- A. Dike-rock-tunnel interactions

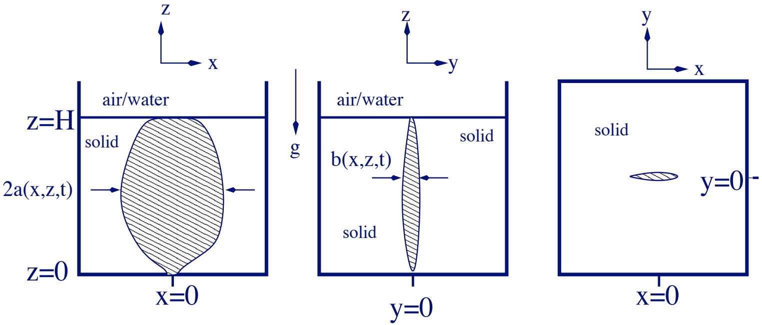
1. Motivation

- A visco-elastic solid loaded under gravity with a free surface is subjected to an influx of fluid at depth.
- What happens?
- To be concrete. Consider the following analog experiment, wherein gelatin is the elastic solid and water the intruding fluid:

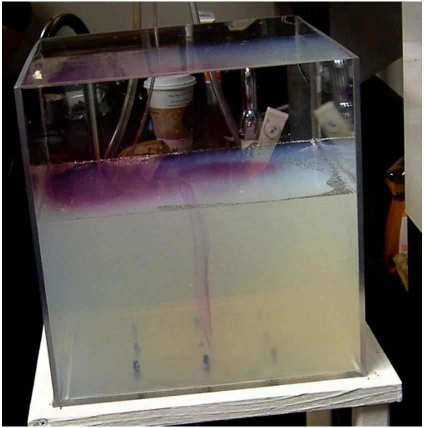


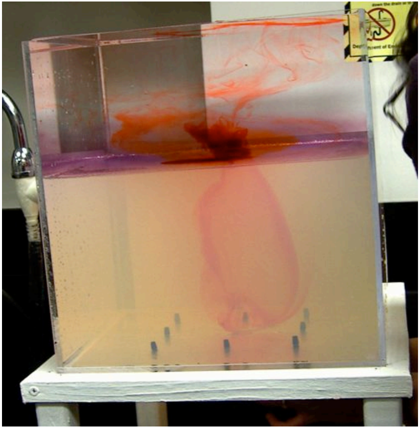
Fluid cavity is thin and leaf shaped

- Under steady influx of fluid a leaf-shaped fluid cavity forms spreading from the localized source at the bottom, and finally narrowing to a wider and shorter crack at the free surface:



. . . fluid cavity thin and leaf shaped . . .





Why are experiments and modeling important?

Because of relevance to geology and industry.

Geology:

- magma dike and conduit formation in San Rafael Volcanic field, and
- Shimabara-Beppu Graben: dike-rock-fault interactions.

Industry:

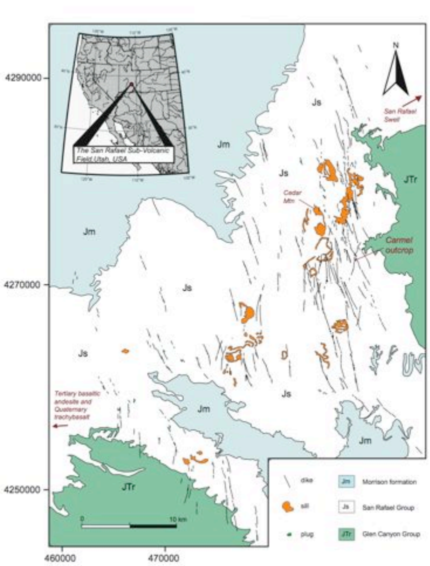
- enhanced oil-recovery,
- CO_2-sequestration,
- mining: dike-rock-tunnel interactions?

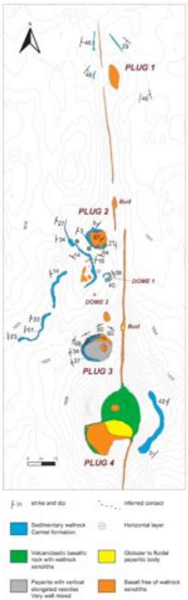
Geology: San Rafael Volcanic fields, Utah

Volcanic plumbing system revealed near Earth's surface:

- 50-500m erosion of soft host rock,
- frozen remains basaltic dike-conduit system,
- courtesy Mikel Diez (Ph.D. thesis, USF).

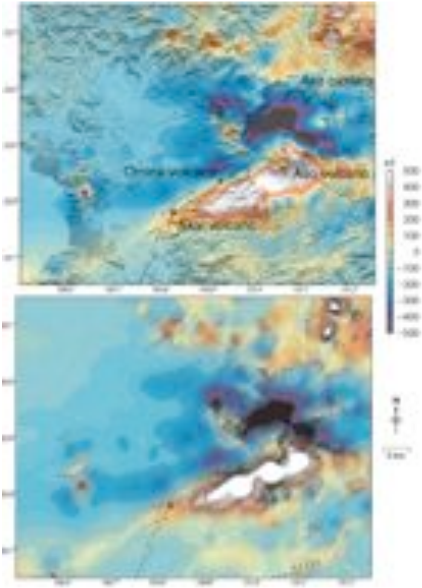
Geology: San Rafael Volcanic fields, Utah





Geology: Shimabara-Beppu Graben

- Dike-rock-fault interactions, Kyushu island, Japan
- Why alignment of volcanoes along fault?



Challenges

Accurate measurements:

- of elastic parameters required: elastic modulus, fracture strength;
- of overpressure, sample height; needed to assess their influence.

We will show conceptual measurements and discuss their ramifications.

Mathematical modeling:

- outline elasto-hydrodynamical (numerical) model;
- used DG finite element discretization (e.g., research Twente B. & Jaap van der Vegt).

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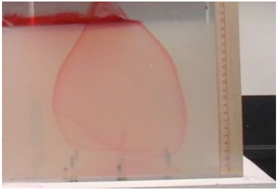
2. Conceptual experimental results

Variations in dike morphology:

- a. flow field in single dike,
- b. dike coalescence and dike-fault interactions,
- dike-rock-tunnel interactions.

a. Flow field in single dike

. . . flow focusing in central channel with side leafs . . .



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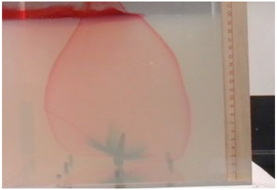

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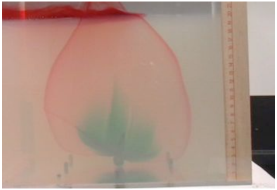

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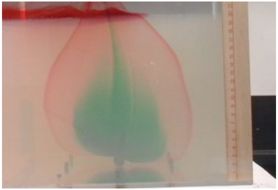

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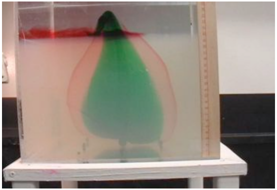

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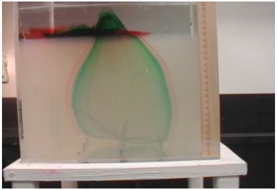

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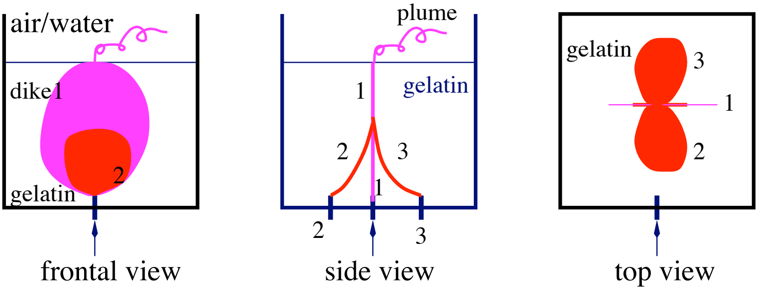
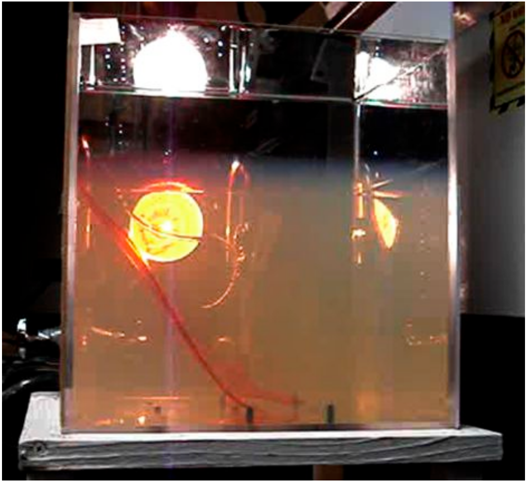
b. Dike coalescense and dike-fault interactions

Cf. Takada's work (1994abc): without differential stress primary dike attracts new dikes. Sketch of observations:



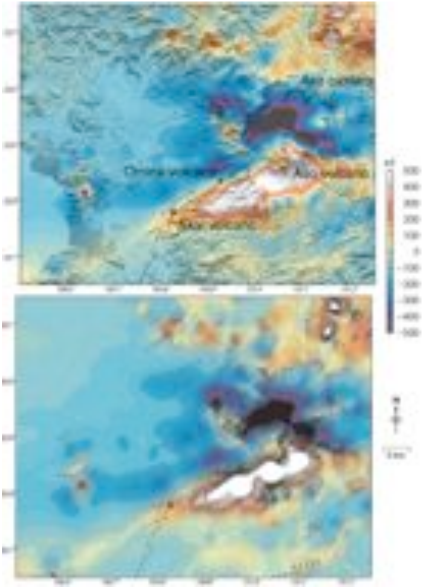
. . . dike-fault interactions

Hypothesis: gel had separated from the front of the tank to form a "fault", during transport, and attracted the dike.

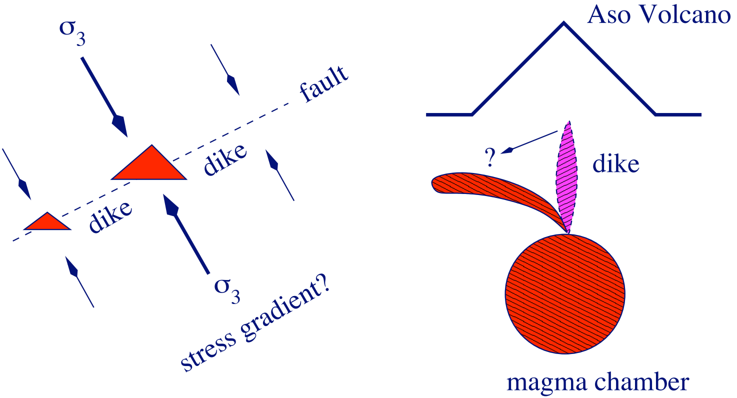


. . . dike-fault interactions: Aso Volcano

- Dike-rock-fault interactions, Kyushu island, Japan
- Why alignment of volcanoes along fault?



Hypothesis: differential stress low at the time of dike injection?



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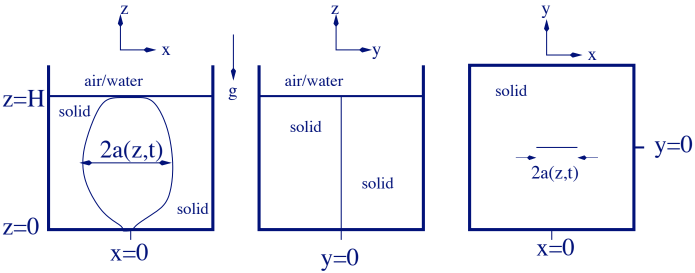
3. Elasto-hydrodynamic lubrication model

Coupling of models:

- Linear 3D elasticity: displacements $u_i = (u_1, u_2, u_3)$.
- Width-averaged viscous fluid using lubrication theory: effective pressure P.
- Matching at dike walls of stress in solid with pressure P.
- Minor $b(x, z, t)$ and major $a(z, t)$ axes of dike.

Effective domain due to linearization and lubrication:



...elasto-hydrodynamic model ...

- Static elastic eqns (relative to lithostatic); $u_i = u_i(x, y, z)$:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\mu\left(u_{i,j} + u_{j,i}\right) + \lambda\, u_{k,k}\,\delta_{ij}\right) = 0 \qquad (1)$$

- Nonlinear advection-diffusion fluid model;
  $b = b(x, z, t)$, $P = P(x, z, t)$:

$$\partial_t b - \nabla_{xz} \cdot \left(\frac{b^3}{12\,\mu_f}\,\nabla_{xz} P\right) = 0 \qquad (2)$$

- At effective dike boundary: $\sigma_{22} = -P$; thus
  $b(x, z)/2 = u_2(x, 0, z) = \mathcal{L}(P)$; linear integral operator $\mathcal{L}(\cdot)$.
- BC's: over- and atmospheric pressure.

. . . determine unknown dike geometry

"Magmatic" advection-diffusion equations:

- Nonlinear integro-partial-differential equation:

$$\partial_t b - \nabla_{xz} \cdot \left(\frac{b^3}{24\,\mu_f} \nabla_{xz} \mathcal{L}^{-1}(b) \right) = 0; \qquad (3)$$

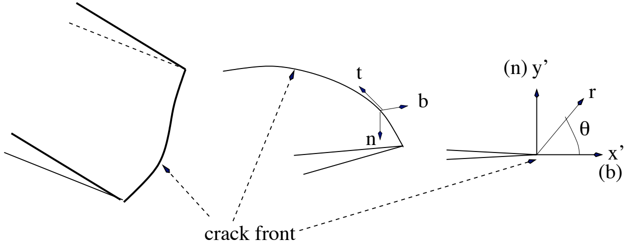
  cf. Lister (1990); extending Pinel & Jaupart (2000), B. et al. (2005), Woods et al. (2006).

- Geometry dike not fixed: unknown major/minor axes $a(z)$ and $b(x, z, t)$.
- Use shooting method: estimate $a(z)$, solve equation (3) till steady state $b(x, z)$.
- Check failure criterion at $x = \pm a(z)$, re-estimate $a(z)$, et cetera . . . convergence to threshold of failure at $x = \pm a(z)$.

- Stress intensity K_I at crack front (cf. Ingraffea et al.):

$$v' = 2(1+\nu)\frac{K_I}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}(2-2\nu-\cos^2\frac{\theta}{2}) \qquad (4)$$

displacements v' in y'–direction (mode I).



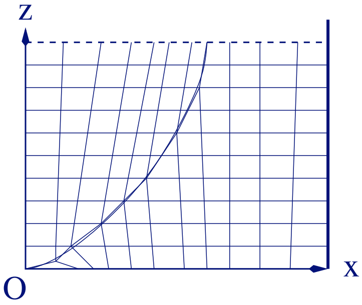
. . . numerical algorithm

A (dis)continuous Galerkin finite element method:

- Calculate $P = \mathcal{L}^{-1}(b/2)$ and advance b in time.
  - $\sqrt{r}$–behavior at crack front,
  - flux $Q = b^3 \partial_z P$ at origin (point source); $b, \Delta P \propto z^{1/4}$.
- Calculate $K_I(z,t)$ along $(x,z) = (a(z,t),z)$.
- If $K_I > K_I^c$ advance front, and vice versa.
- Continue . . . till $|K_I - K_I^c| < \epsilon$, or till steady state is reached.

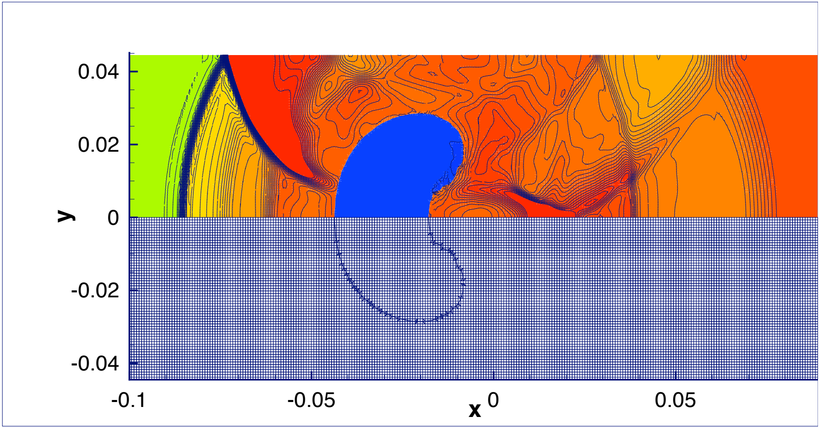
Galerkin FEM; matching meshes solid and fluid:

- Steady case: topology mesh fixed, (un)structured



Discontinuous Galerkin FEM; matching meshes solid and fluid:

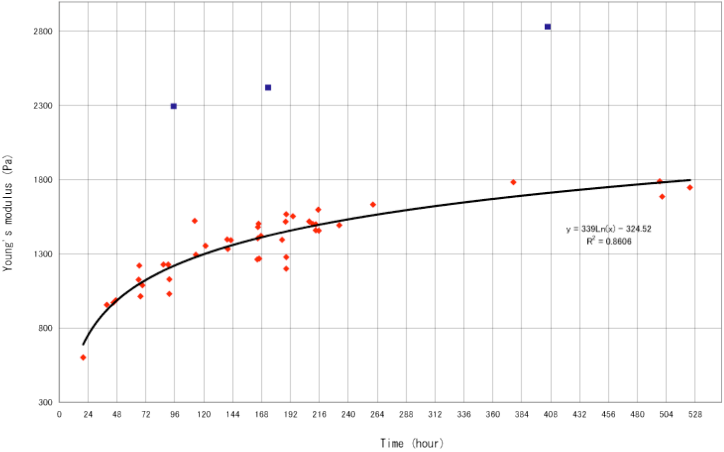
- Dynamic topology, two-fluid mesh (Sollie); structured



. . . parameters

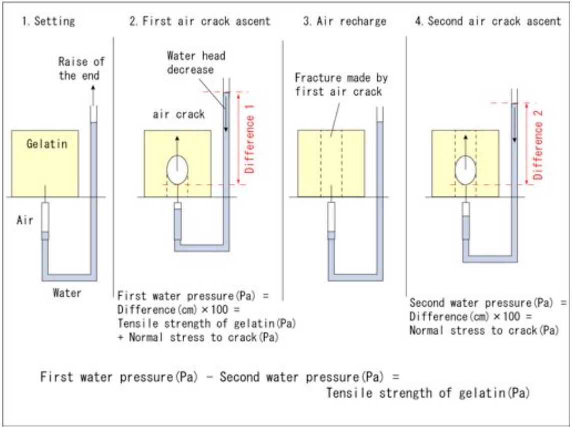
Poisson's ratio $\nu = 0.5\lambda/(\lambda+\mu) \approx 0.5$, and Young's modulus:

- $E = \mu(3\lambda+\mu)/(\lambda+\mu) \approx 2300$Pa changes over time . . .



Global failure criterion?

- Define principal values $\sigma_1 \geq \sigma_2 \geq \sigma_3$.
- Hydraulic fracturing ($\sigma_3 > -\Delta P$), link to K_I^c unclear.
- Obtain ΔP experimentally for gel:



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4. Discussion

Presented:

- conceptual experiments of flow in fluid-filled cracks, and
- elasto-hydrodynamic lubrication model of gelatine-fluid or "magma-rock interactions".

Future work:

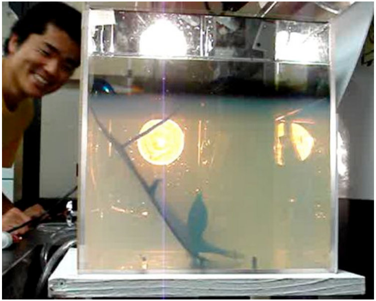
- Validation of single dike experiments versus elasto-hydrodynamic modeling.
- Test elasto-hydrodynamic model at geological scales.
- Check hypothesis dike-fault interactions: lab experiments, support by field measurements.
- Compressible/bubbly multi phase thin-layer fluid flow.
- Dike-rock-tunnel interactions:

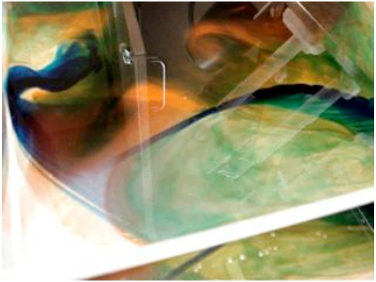
A. Dike-rock-tunnel interactions

- How do we dig a tunnel?

A. Dike-rock-tunnel interactions

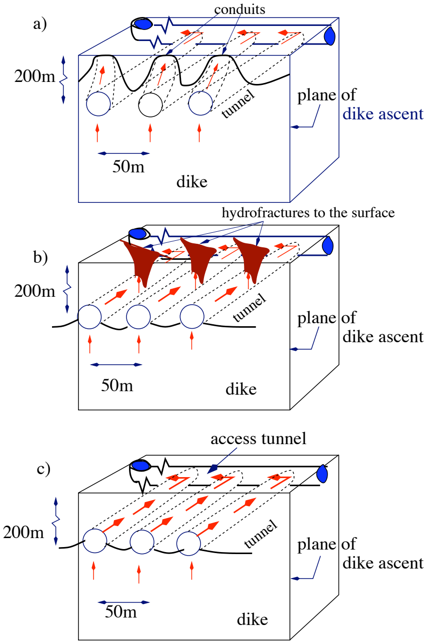
- Tunnel drawn with a straw (before dike formation and after coffee break for the straw).
- Tunnel discharge lowers magma discharge through top dike; dike closes partially.


Side view

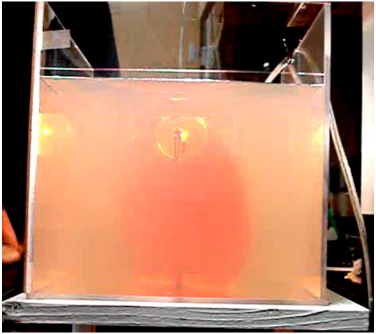

Top view

. . . dike-rock-tunnel interactions

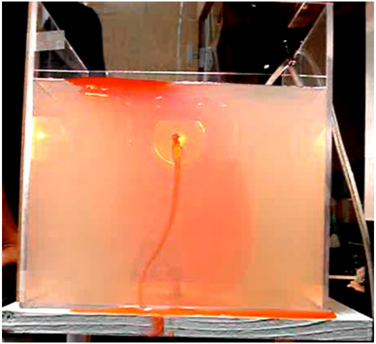
- Tunnel made before dike formation.
- What happens: a, b, or c? (cf. Woods et al. 2002)



a) Dike goes straight through tunnel even though tunnel reduces the discharge at the dike top!



Breakthrough tunnel



After breakthrough at surface

Compressible two-phase fluid

Couple solid dynamics to compressible fluid equations.

- Width-averaged fluid equations; energetics.
- Mass and momentum equations in (x, z)—plane, $\mathbf{v} = (u, w)$:

$$\partial_t(\rho b) + \nabla_{xz} \cdot (\rho b \mathbf{v}) = 0 \quad (5)$$

$$\partial_t(\rho b \mathbf{v}) + \nabla_{xz} \cdot (\rho b \mathbf{v}\mathbf{v}) + b \nabla_{xz} p = -\rho b g \,\hat{\mathbf{z}} + \mathbf{D} \quad (6)$$

- Bubbly pseudo two-phase fluid with isothermal EOS:

$$\frac{1}{\rho(p)} = \frac{n(p)RT}{p} + \frac{1 - n(p)}{\sigma} \quad (7)$$

volatile fraction $n(p) = n_0 - s_H\, p^{\beta}$; constant R, n_0, s_H, β, T.

- Viscous/turbulent dissipation $\mathbf{D}$, e.g. constant ρ and $\mathbf{D} \propto \mathbf{v}$.

Some literature

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